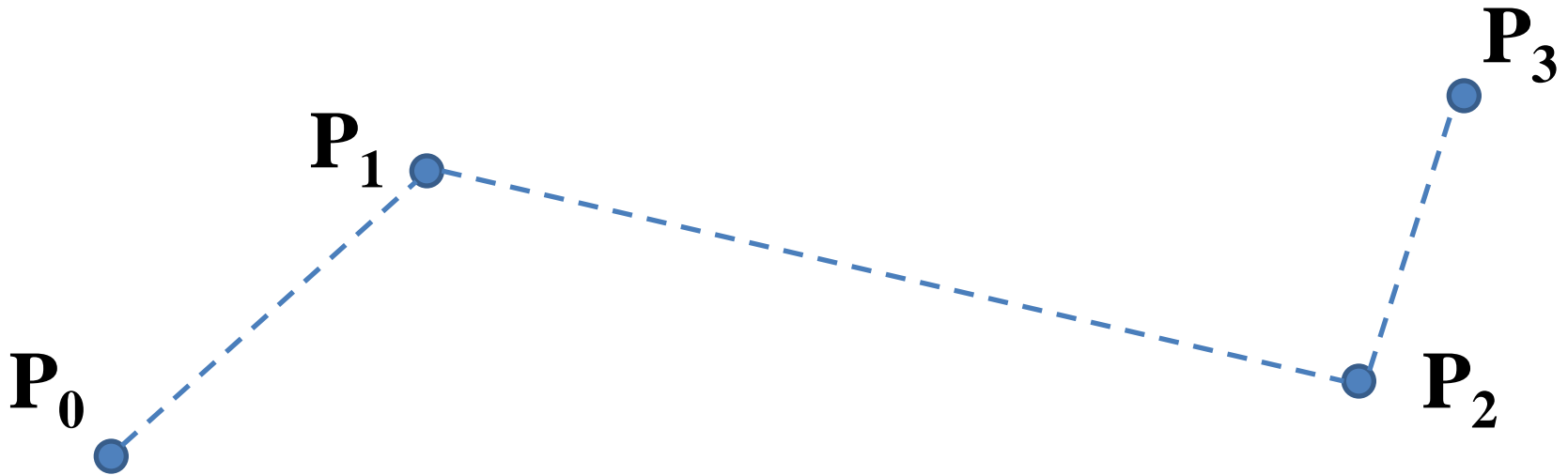


# On the Parameterization of Catmull-Rom Curves

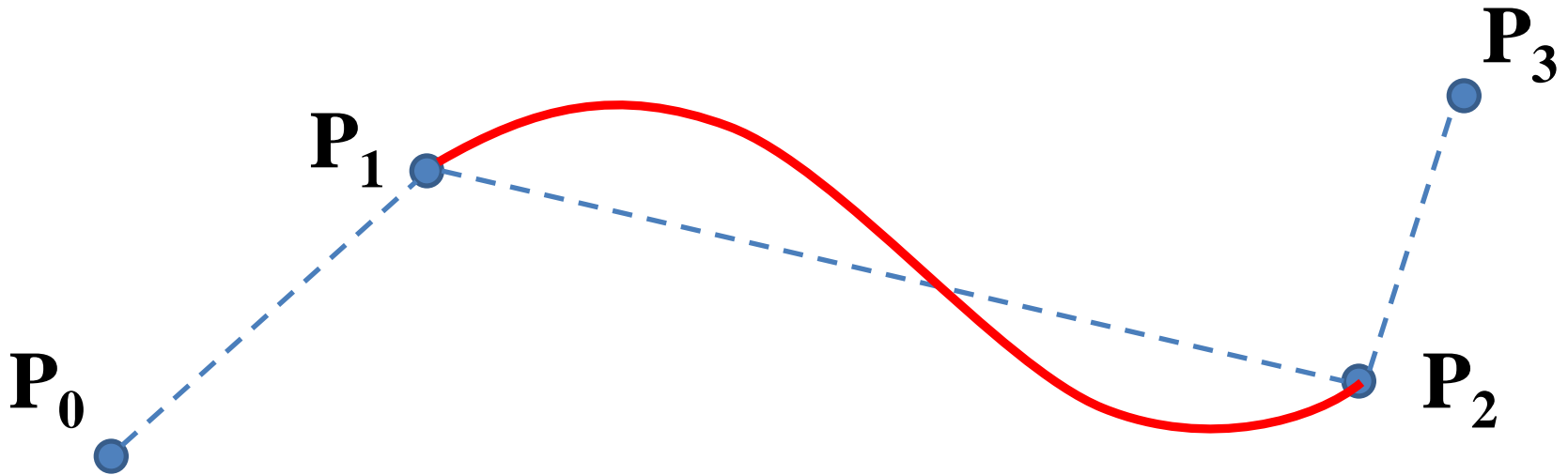
Cem Yuksel   Scott Schaefer   John Keyser

Texas A&M University

# Catmull-Rom Curves

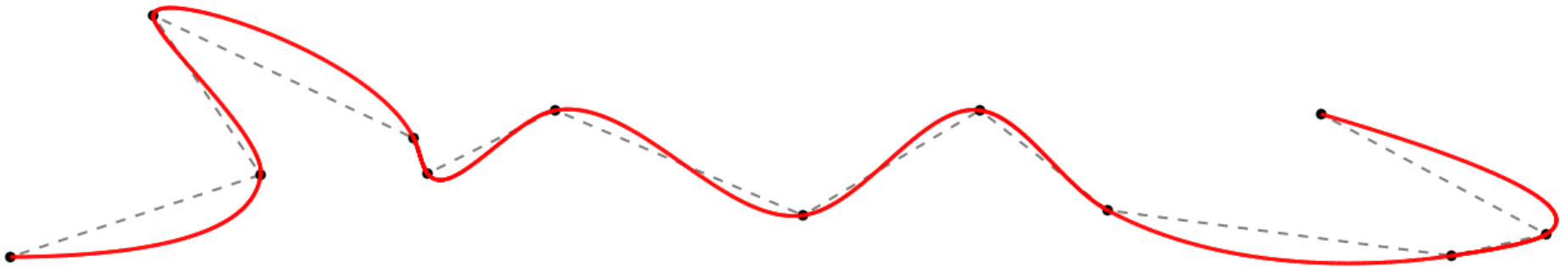


# Catmull-Rom Curves



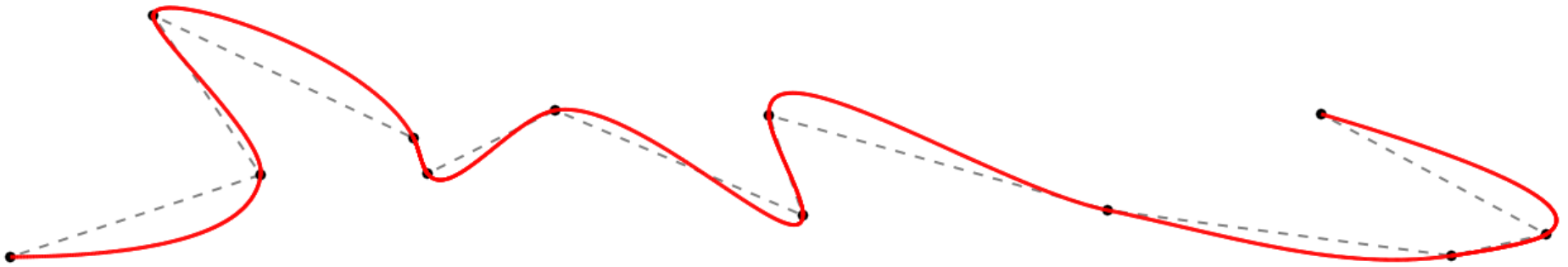
# Catmull-Rom Curves

- Important Properties
  - Interpolate control points
  - Local support
  - Piecewise polynomial representation

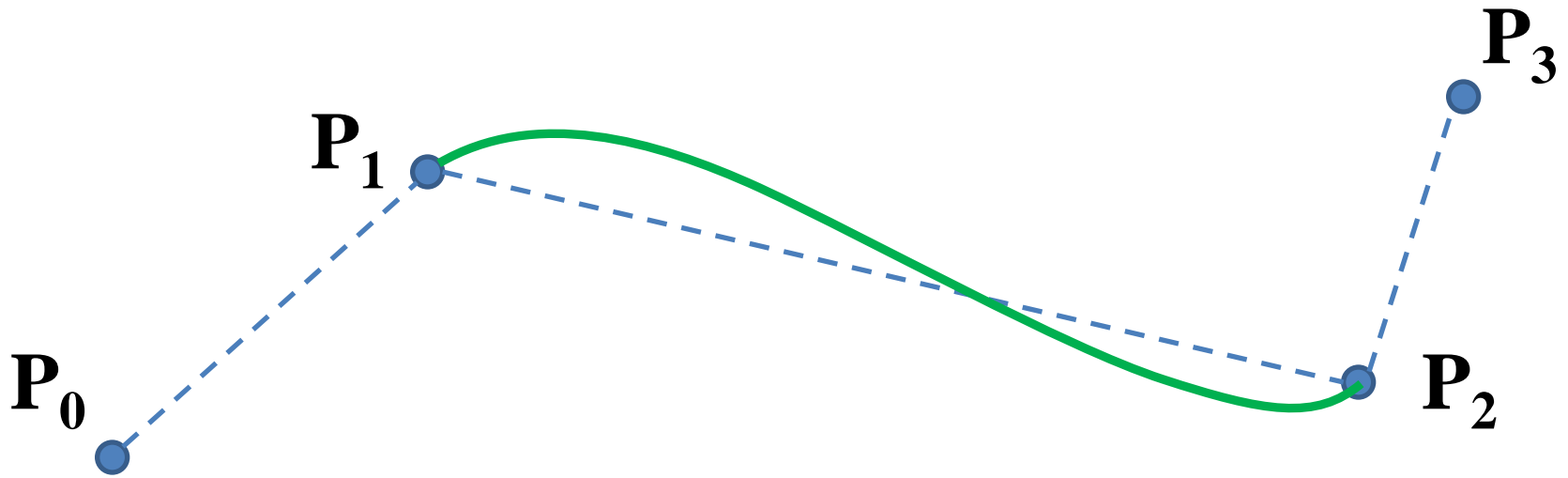


# Catmull-Rom Curves

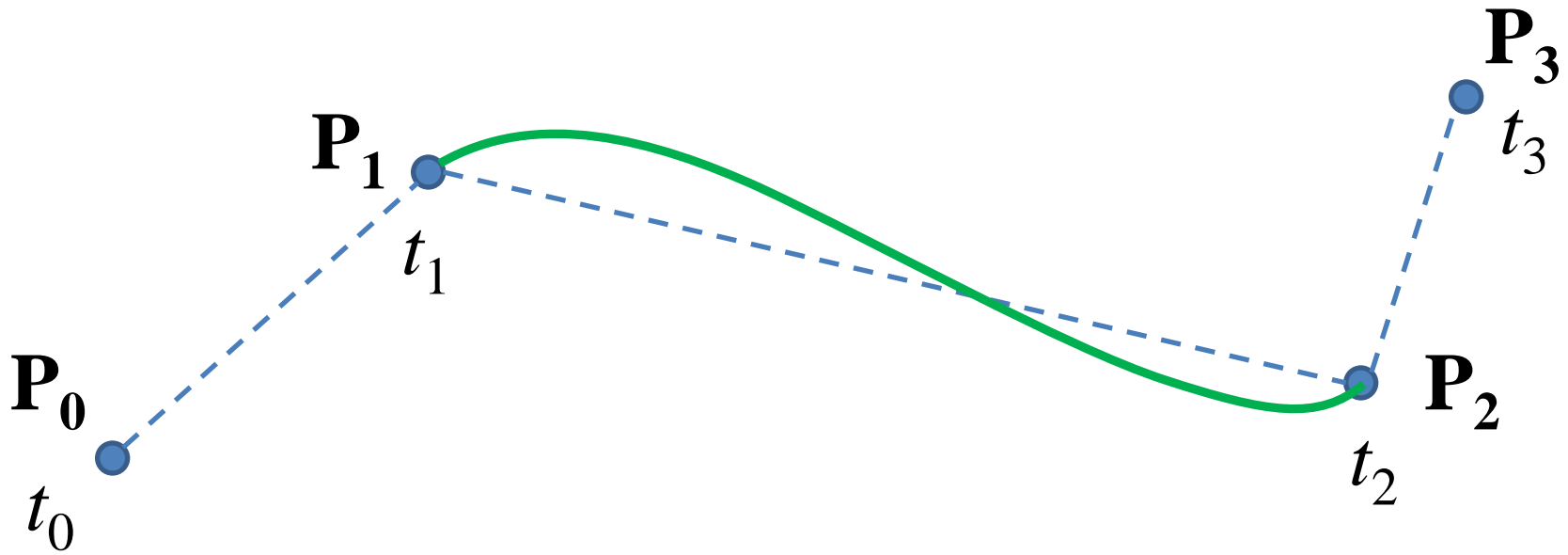
- Important Properties
  - Interpolate control points
  - Local support
  - Piecewise polynomial representation



# Catmull-Rom Curves

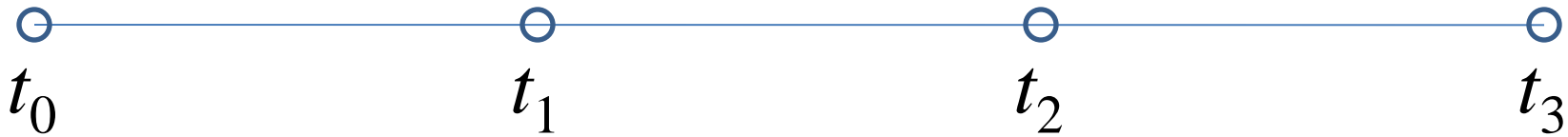
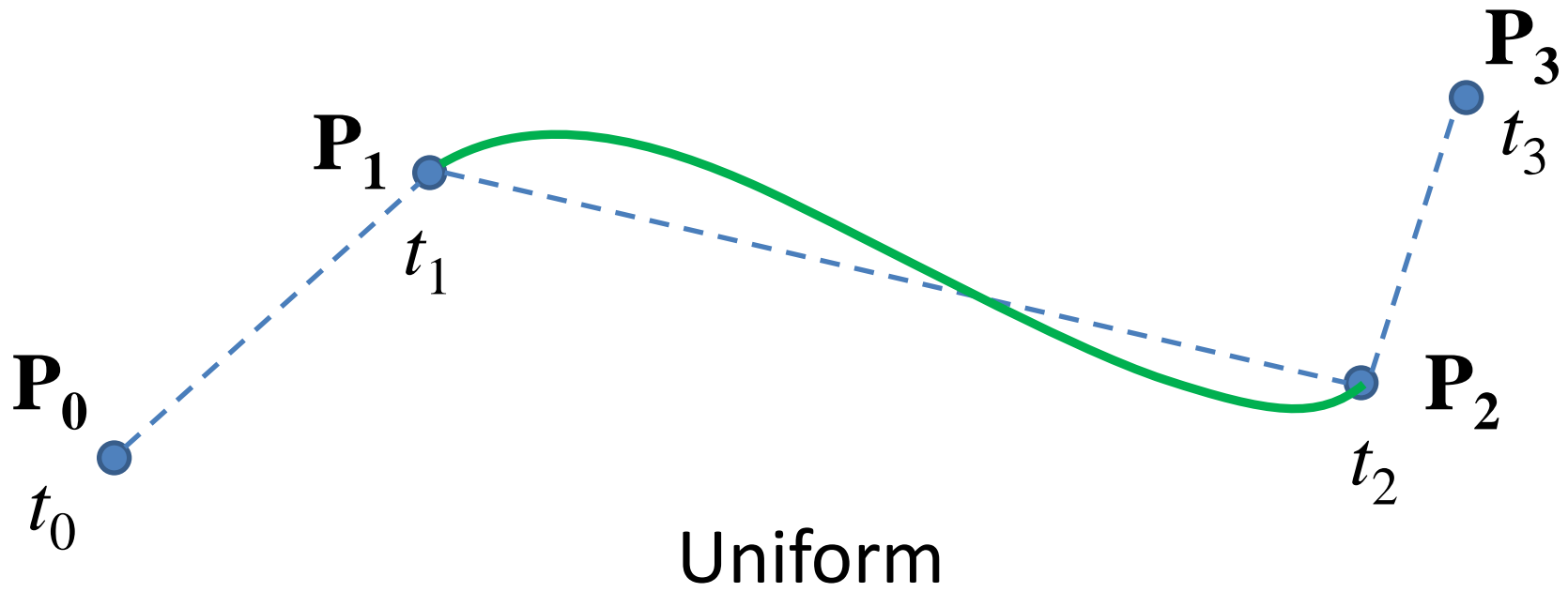


# Catmull-Rom Curves



$$C_{12} ( P_{0,1,2,3}, t_{0,1,2,3} )$$

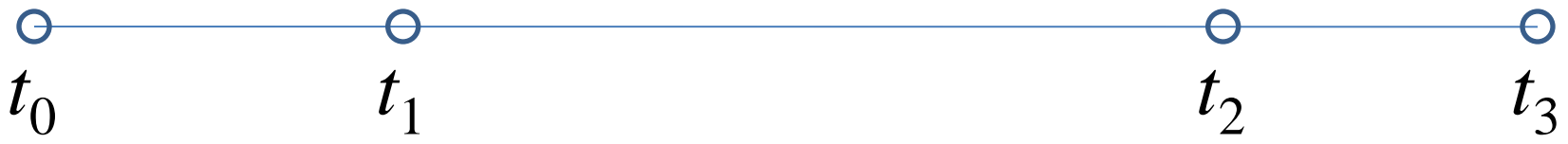
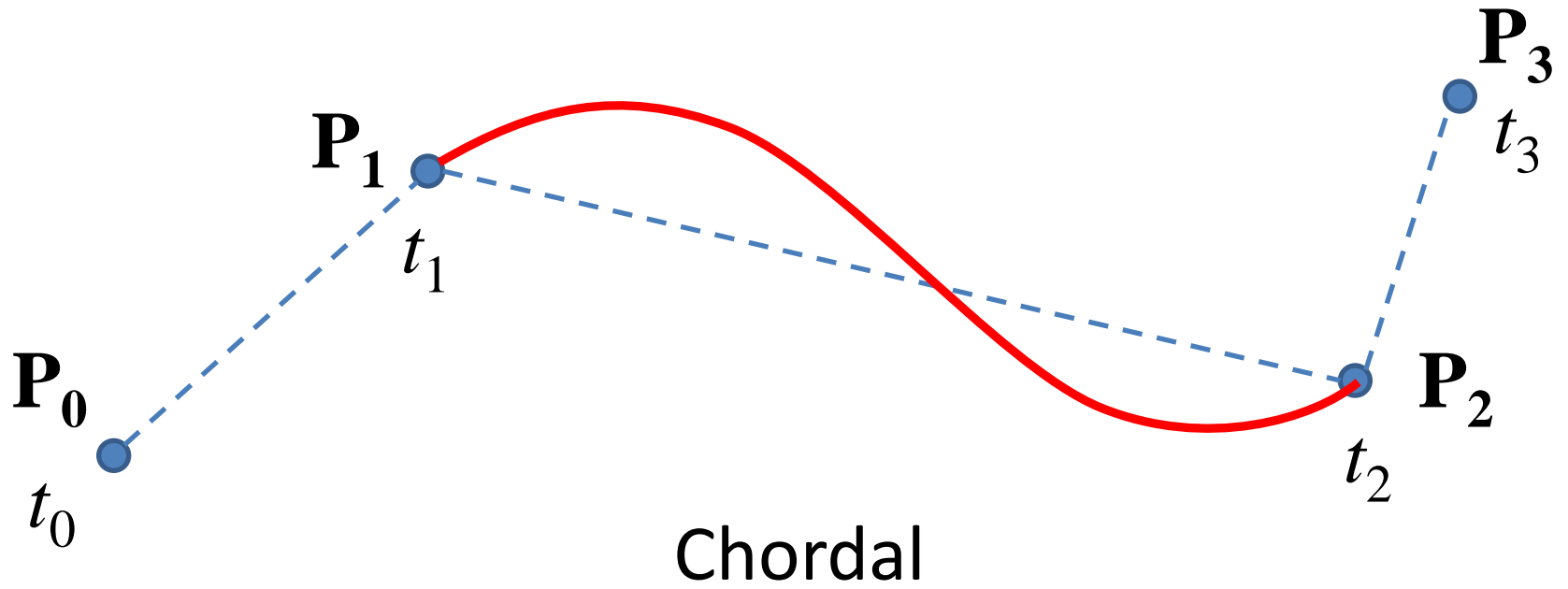
# Catmull-Rom Curves



$$t_{i+1} = t_i + 1$$



# Catmull-Rom Curves



$$t_{i+1} = t_i + | \mathbf{P}_{i+1} - \mathbf{P}_i |$$

# Catmull-Rom Curves

- Parameterization

- Uniform:

$$t_{i+1} = t_i + 1$$

- Chordal:

$$t_{i+1} = t_i + | \mathbf{P}_{i+1} - \mathbf{P}_i |$$

# Catmull-Rom Curves

- Parameterization

- Uniform:

$$t_{i+1} = t_i + |\mathbf{P}_{i+1} - \mathbf{P}_i|^0$$

- Chordal:

$$t_{i+1} = t_i + |\mathbf{P}_{i+1} - \mathbf{P}_i|^1$$

# Catmull-Rom Curves

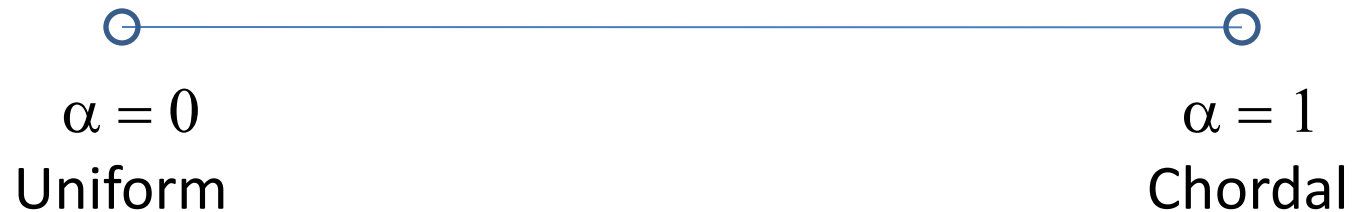
- Parameterization

$$t_{i+1} = t_i + | \mathbf{P}_{i+1} - \mathbf{P}_i |^\alpha$$

# Catmull-Rom Curves

- Parameterization

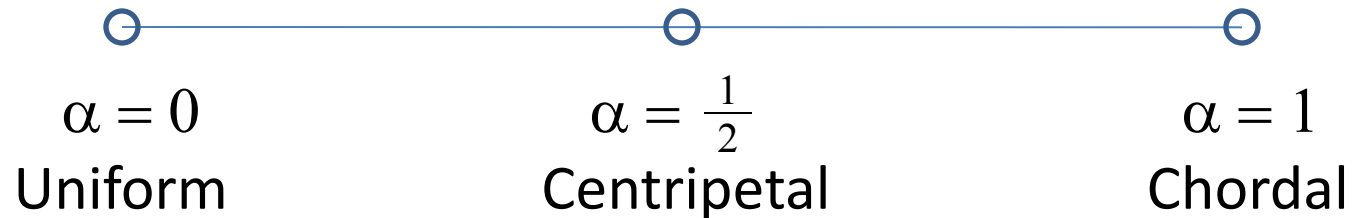
$$t_{i+1} = t_i + | \mathbf{P}_{i+1} - \mathbf{P}_i |^\alpha$$



# Catmull-Rom Curves

- Parameterization

$$t_{i+1} = t_i + | \mathbf{P}_{i+1} - \mathbf{P}_i |^\alpha$$



On the Parameterization of Catmull-Rom Curves

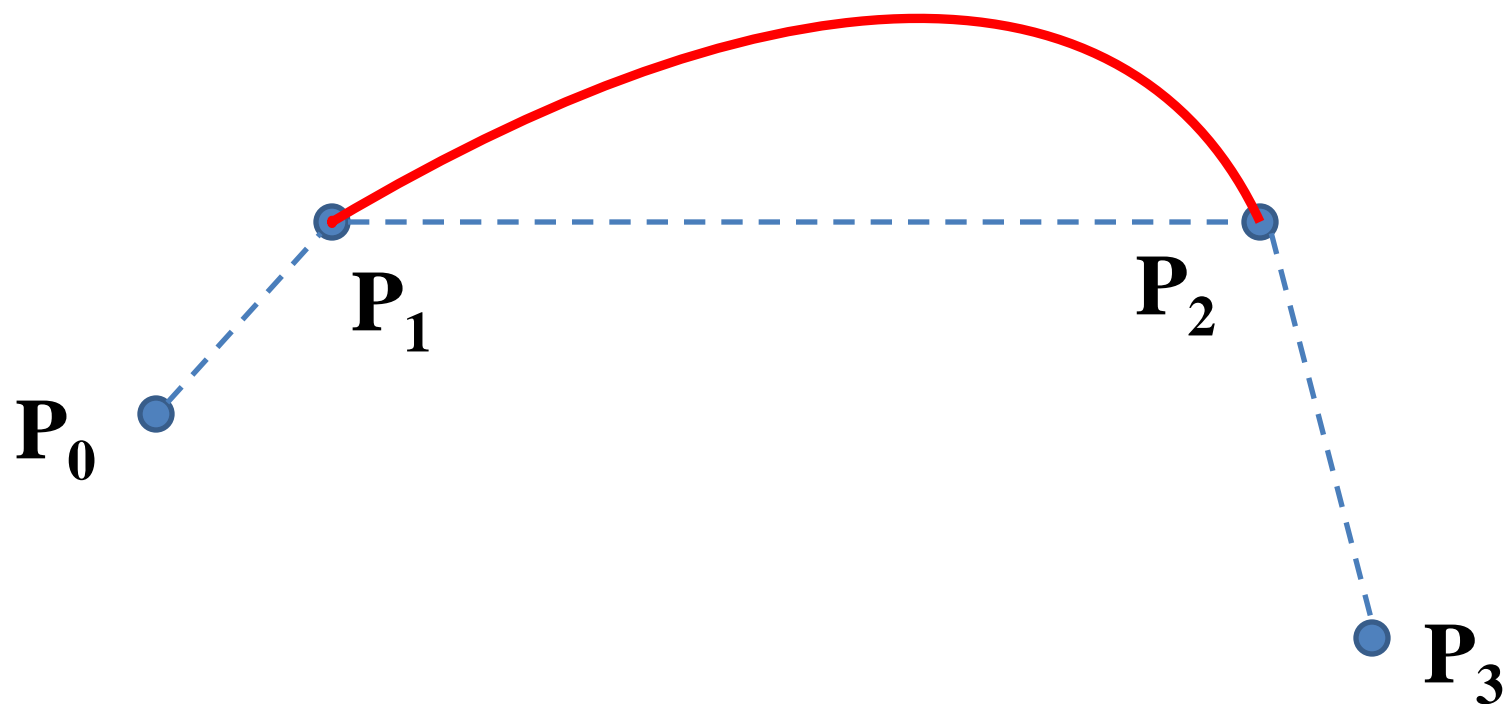
**DEMO**

On the Parameterization of Catmull-Rom Curves

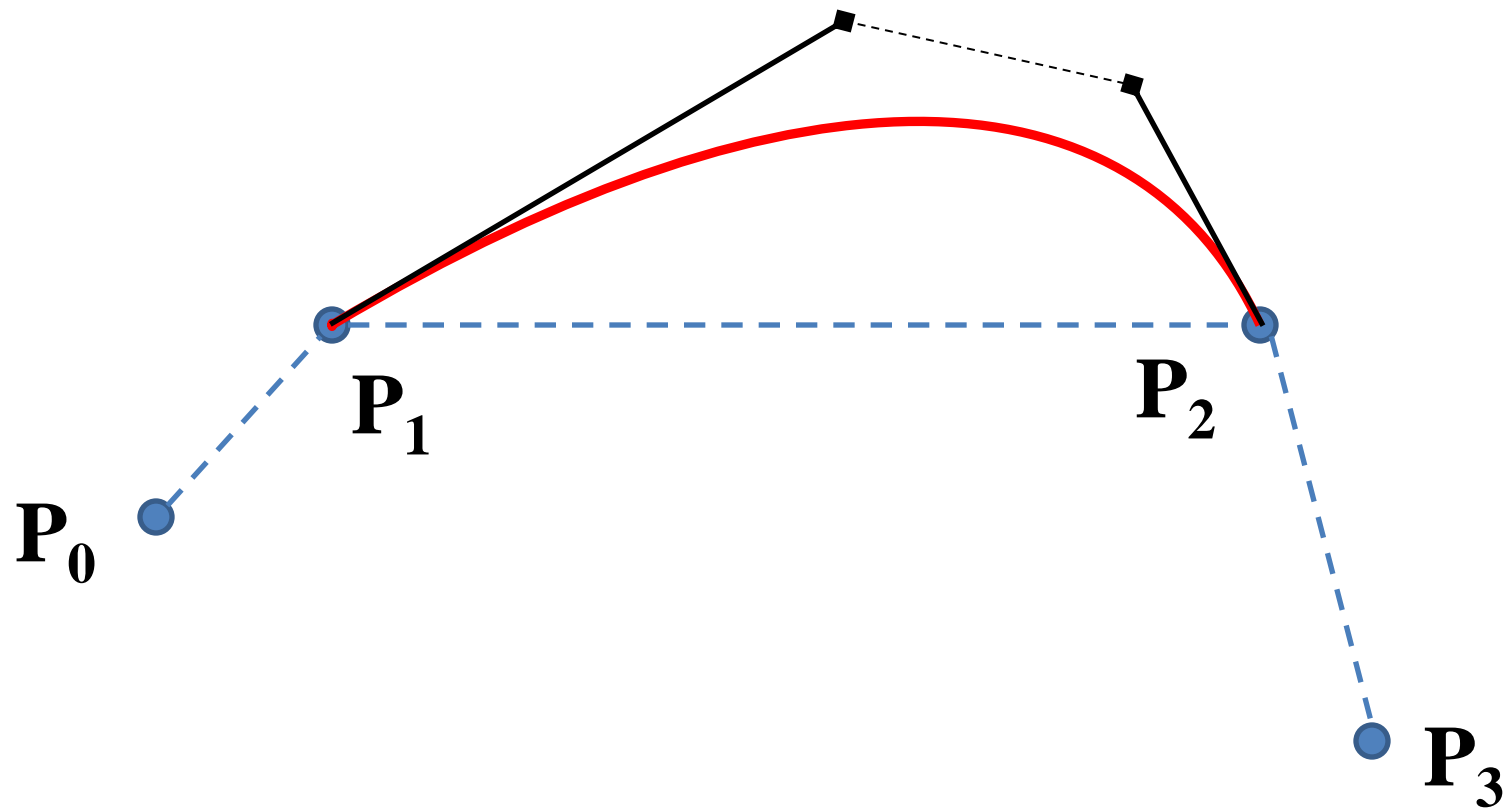
# **CUSPS & SELF-INTERSECTIONS**



# Cusps & Self-Intersections

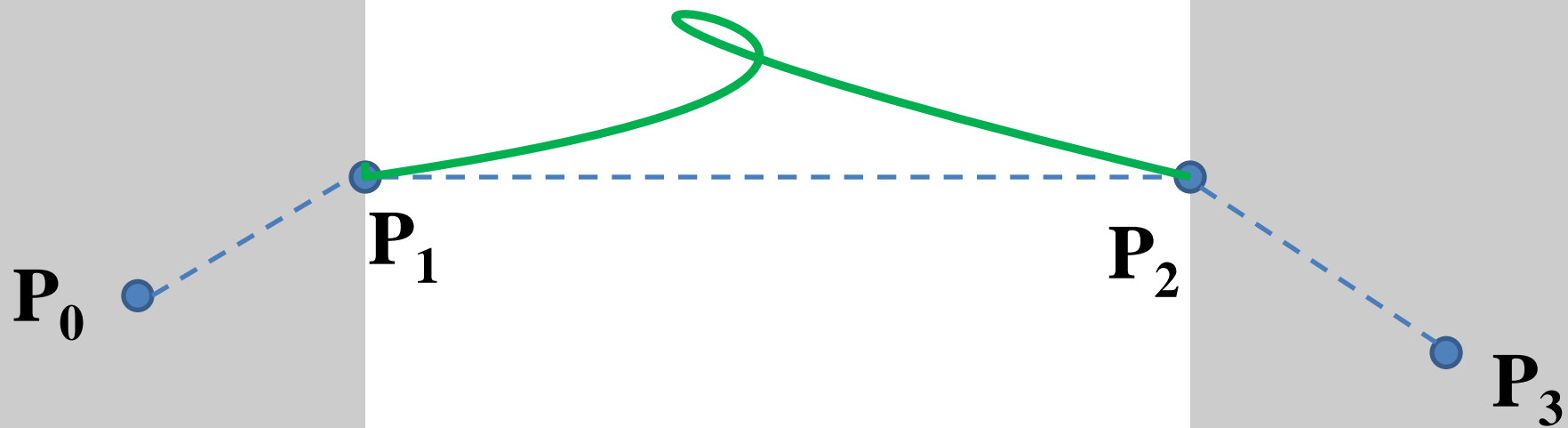


# Cusps & Self-Intersections

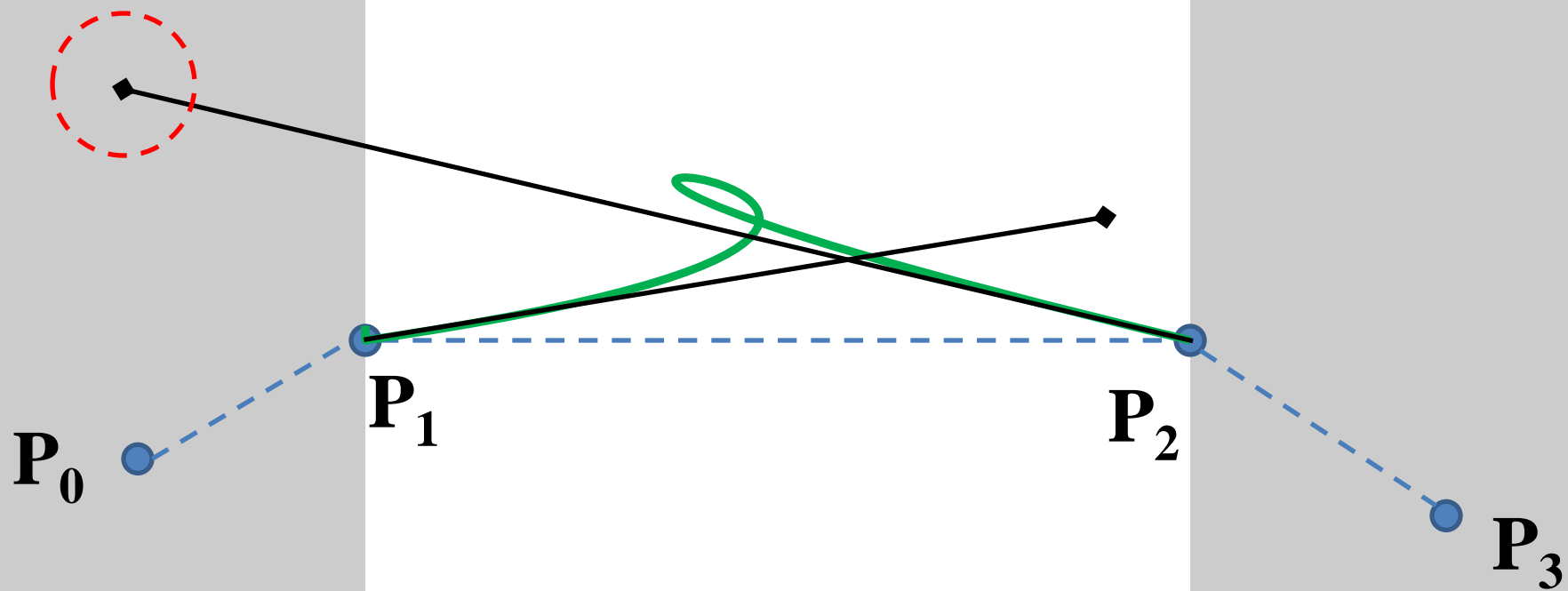




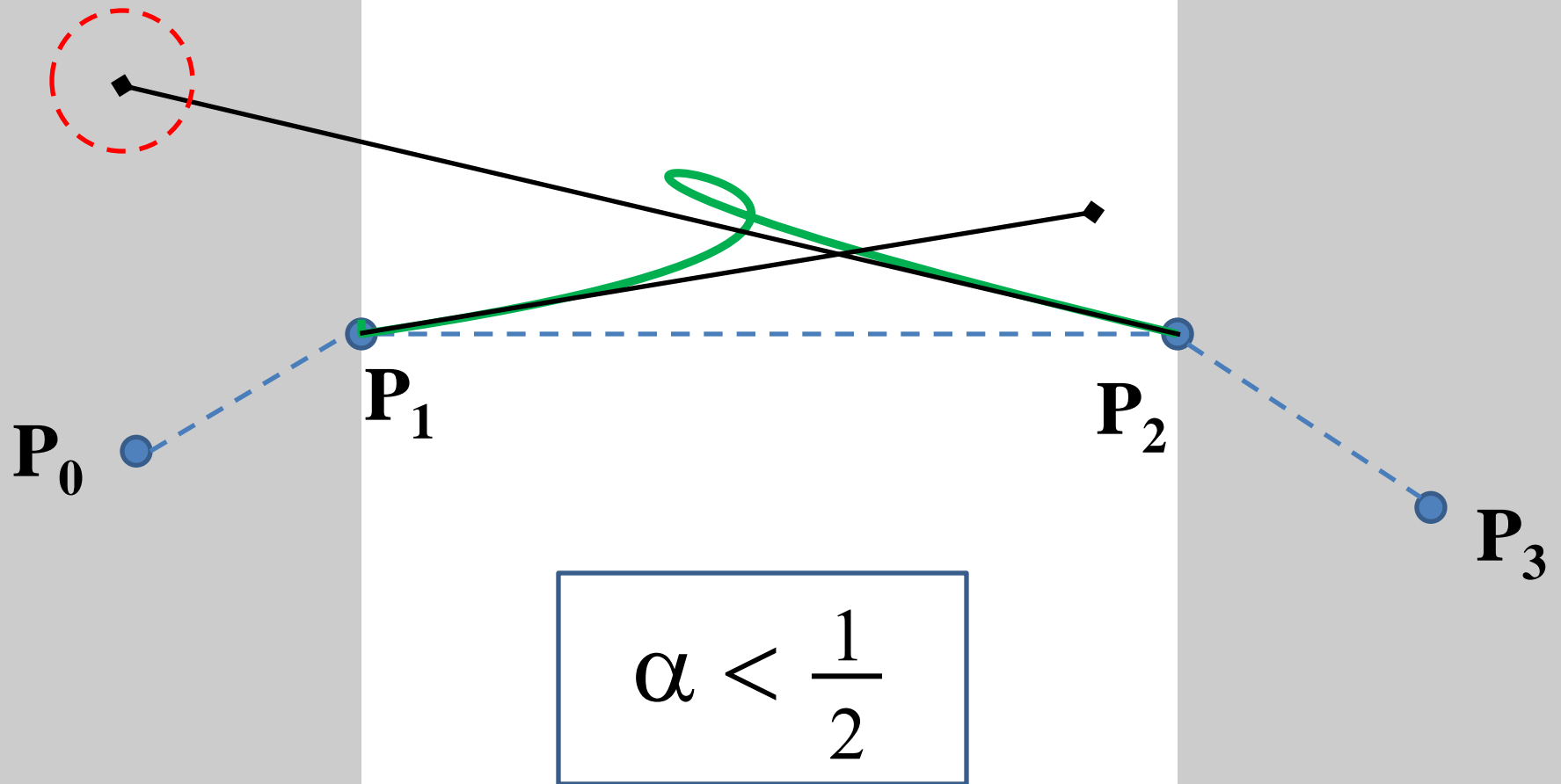
# Cusps & Self-Intersections



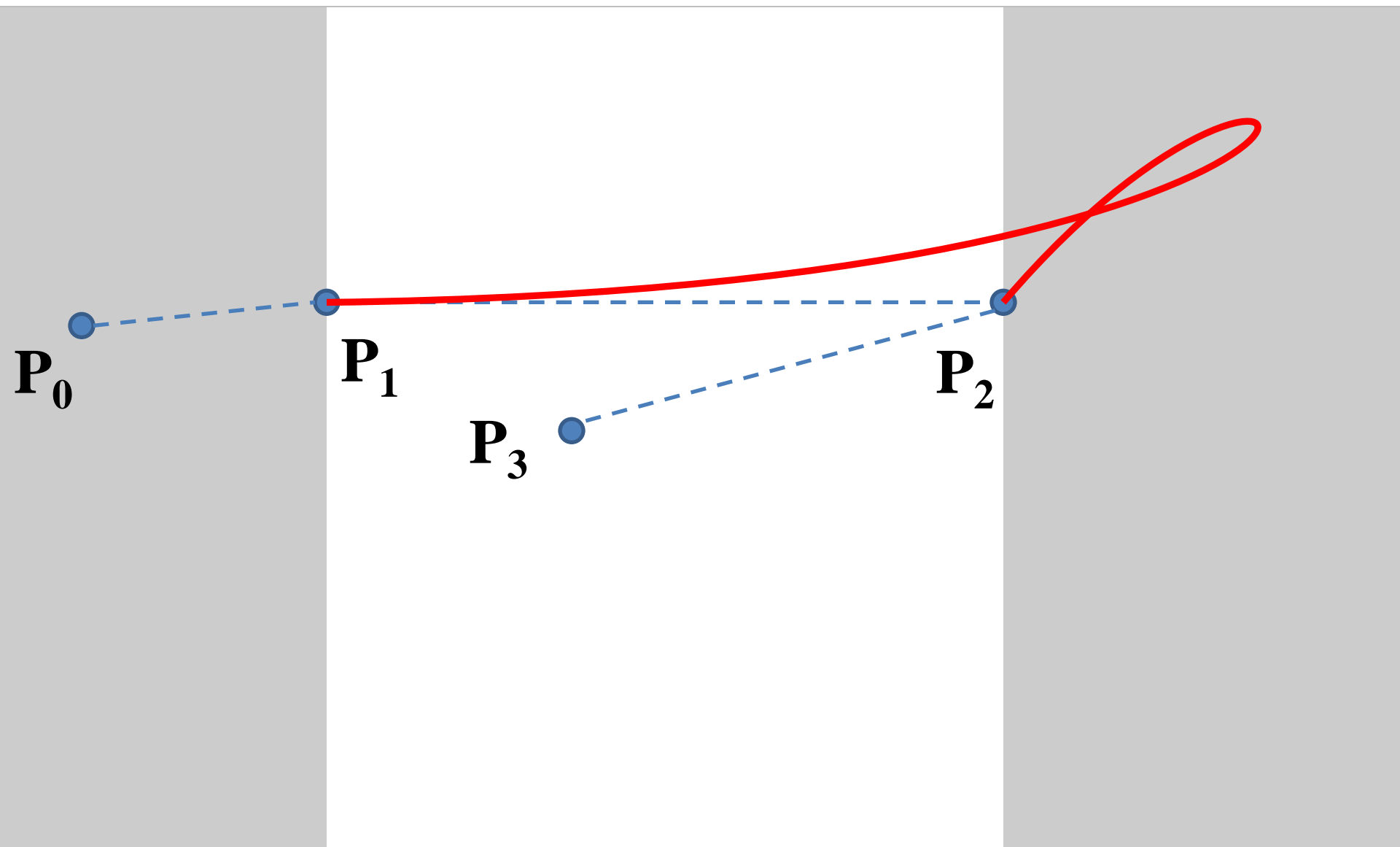
# Cusps & Self-Intersections



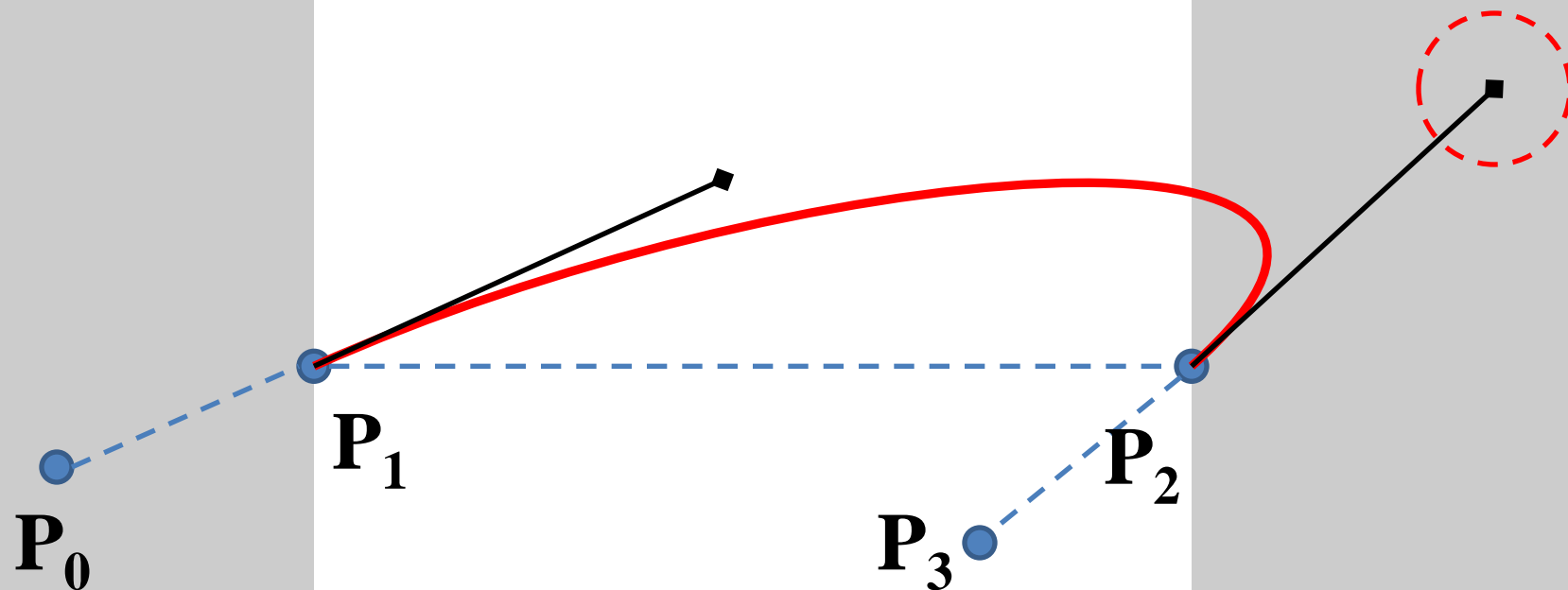
# Cusps & Self-Intersections



# Cusps & Self-Intersections

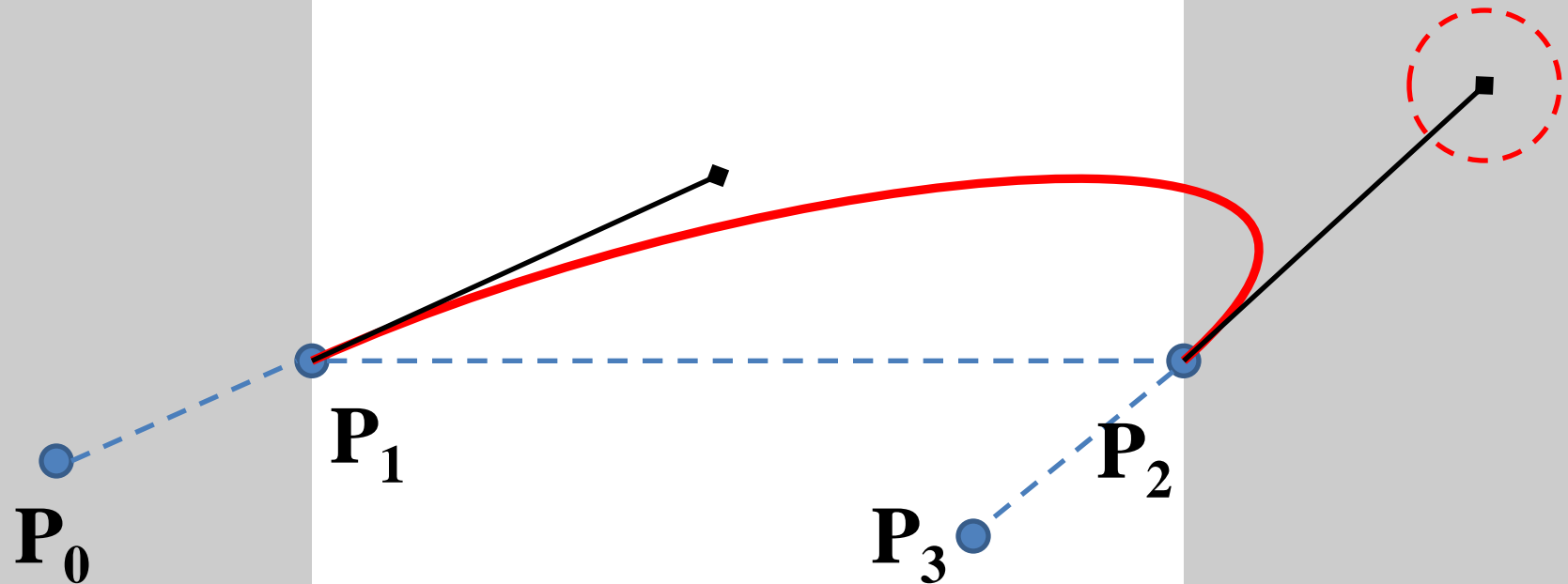


# Cusps & Self-Intersections



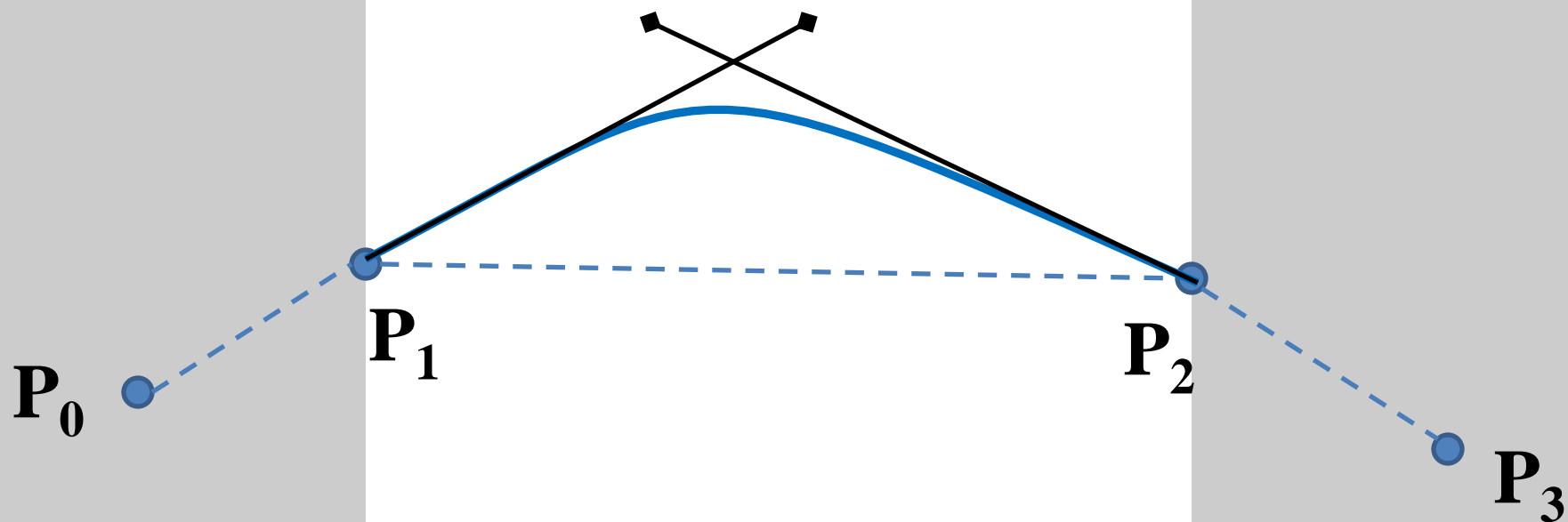


# Cusps & Self-Intersections



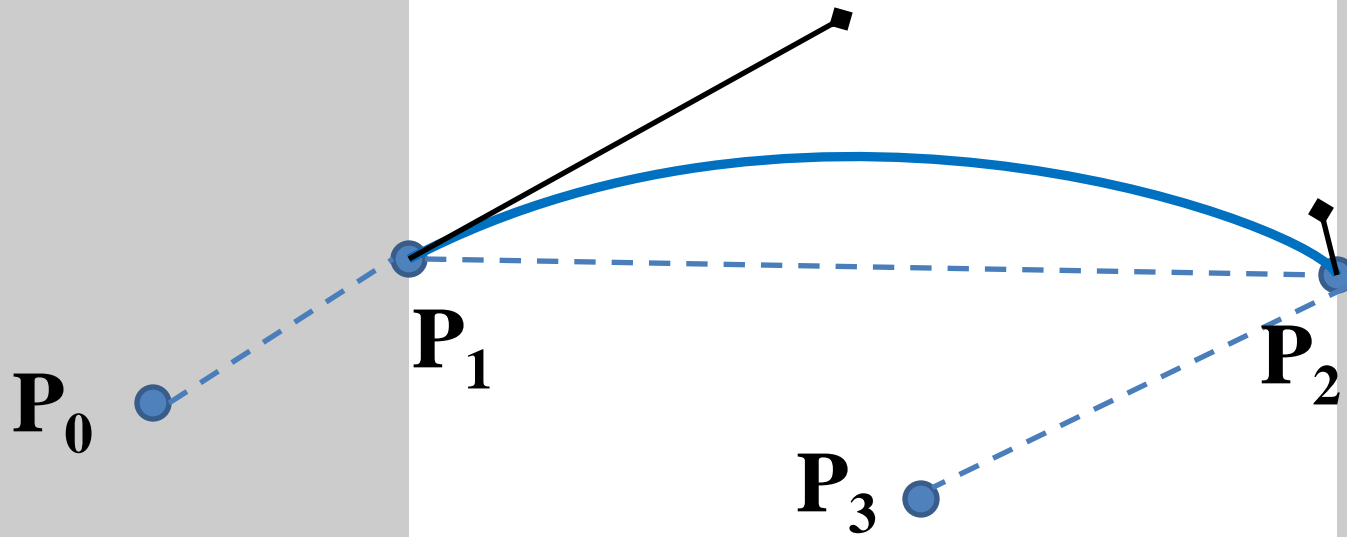
$$\alpha \neq \frac{1}{2}$$

# Cusps & Self-Intersections



$$\alpha = \frac{1}{2}$$

# Cusps & Self-Intersections

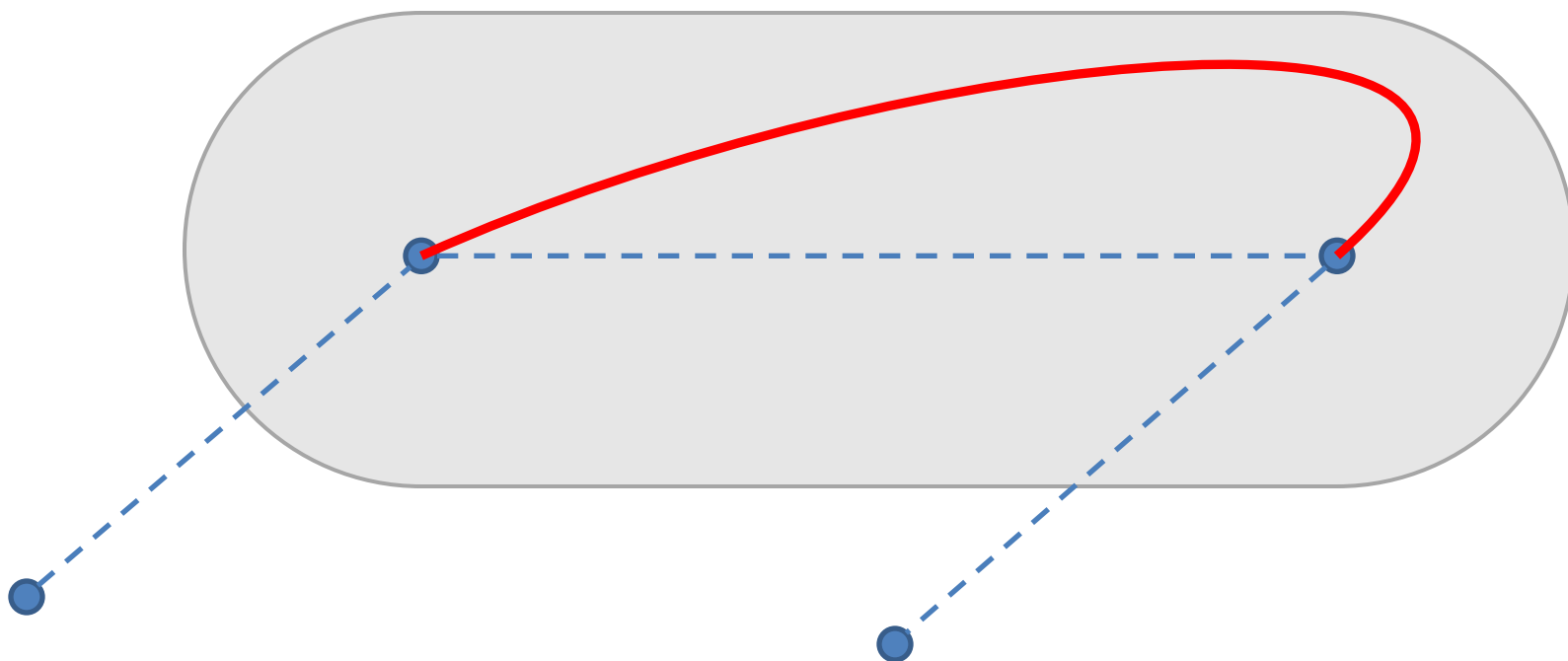


$$\alpha = \frac{1}{2}$$

On the Parameterization of Catmull-Rom Curves

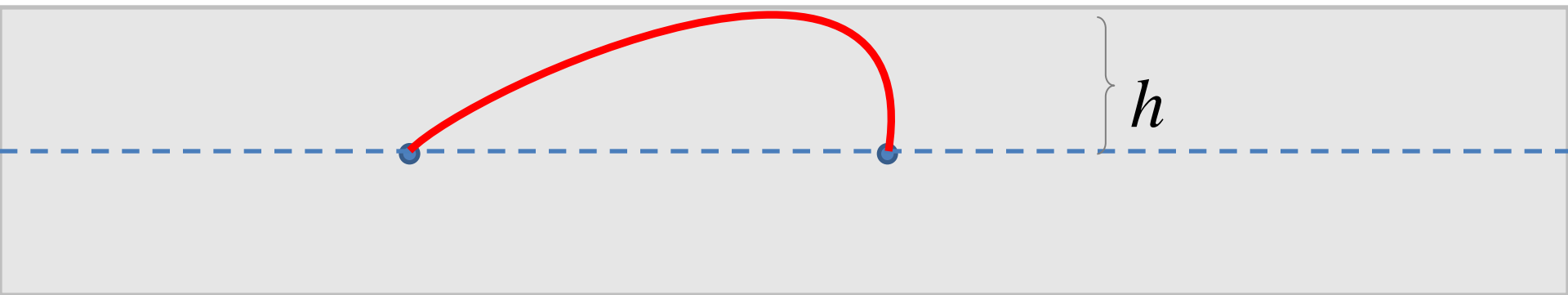
# **DISTANCE BOUND**

# Distance Bound

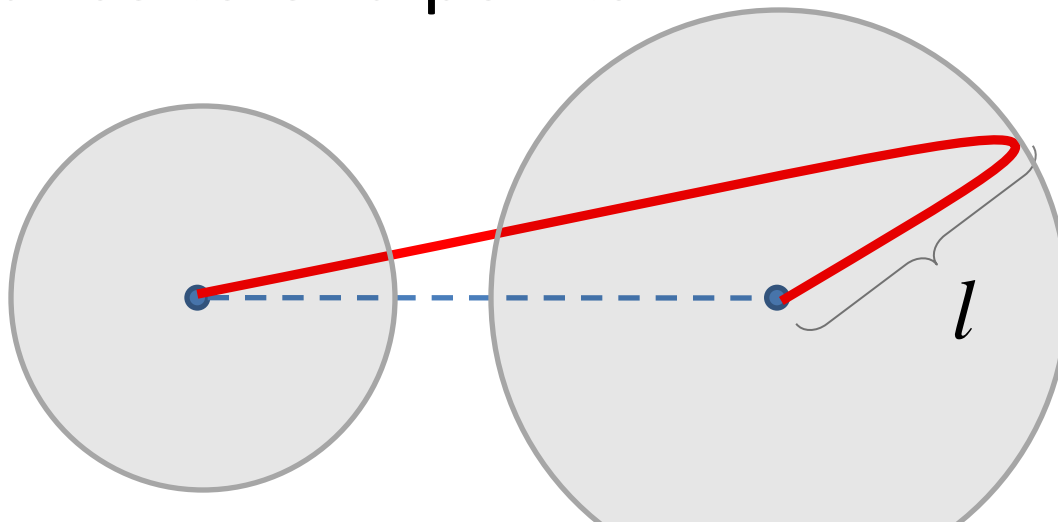


# Distance Bound

- Distance to the infinite line

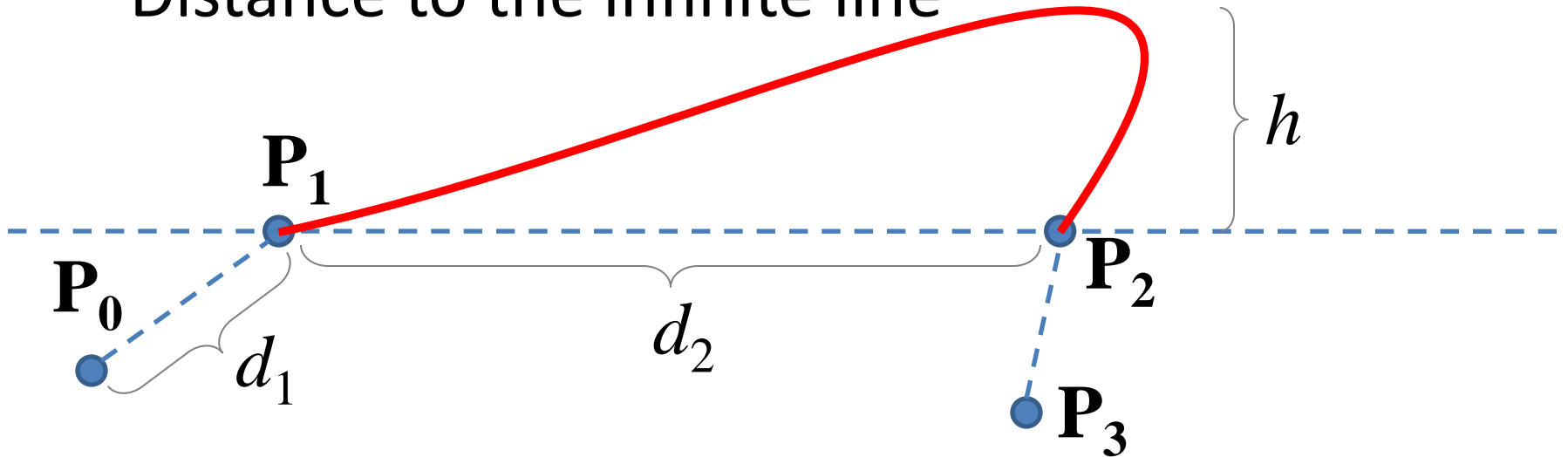


- Distance to end points



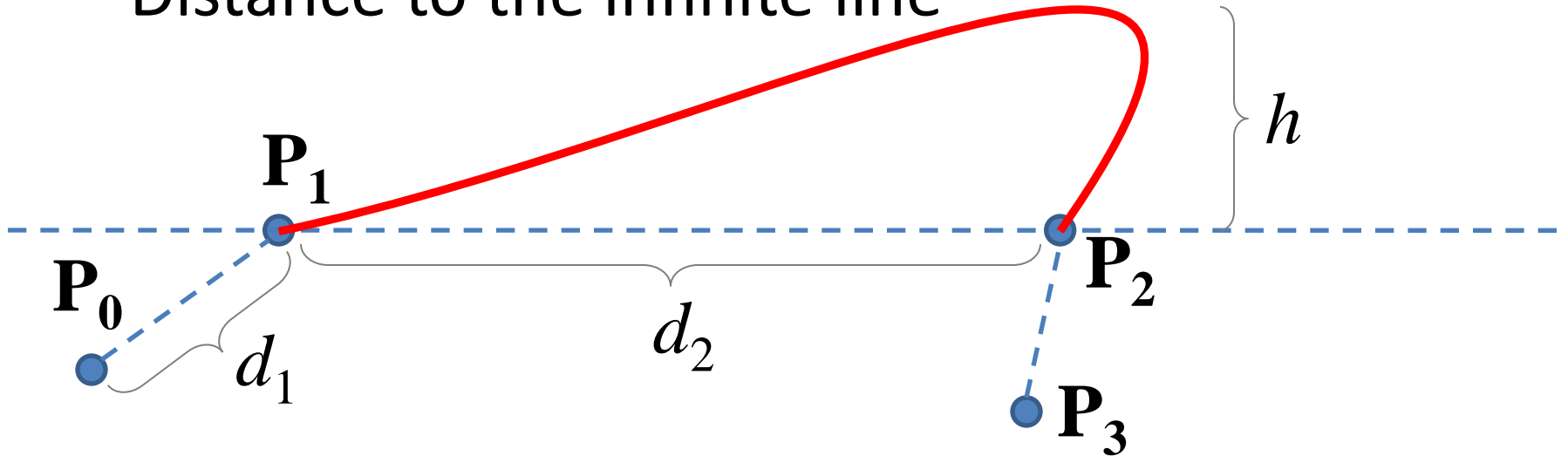
# Distance Bound

- Distance to the infinite line



# Distance Bound

- Distance to the infinite line

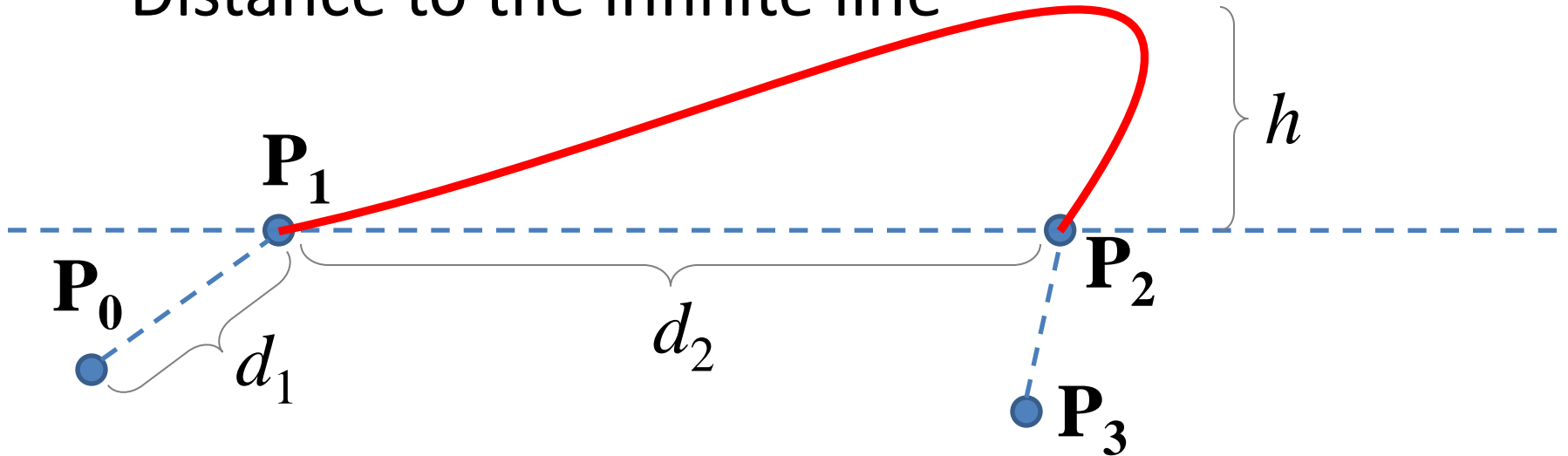


$$h \leq d_2 \times A$$



# Distance Bound

- Distance to the infinite line

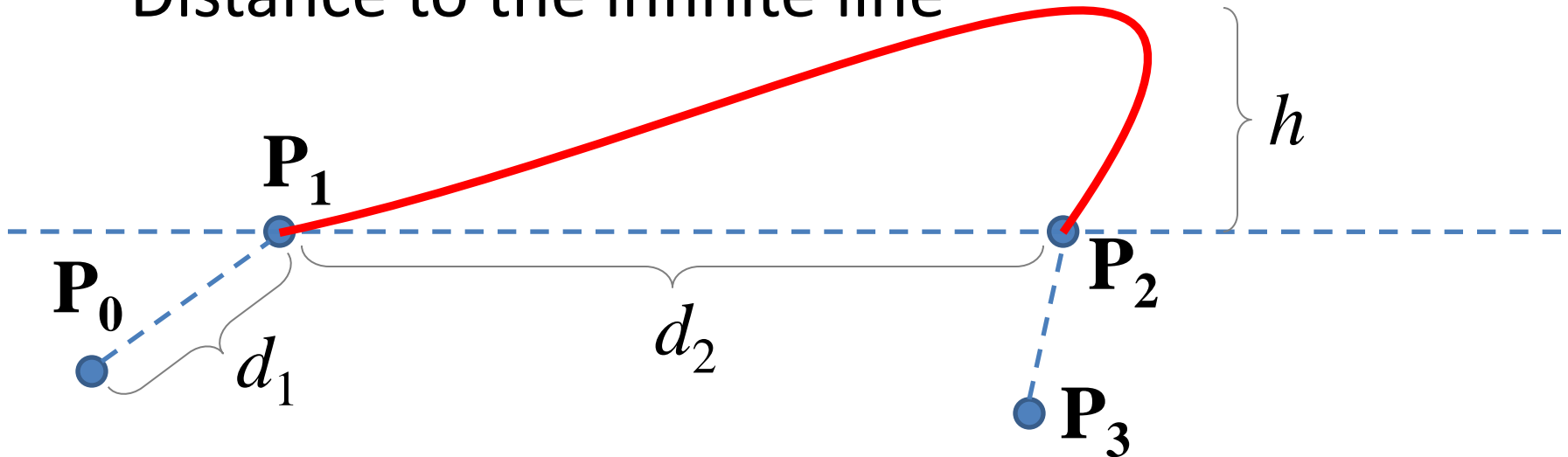


$$r = \frac{d_1}{d_2}$$

$$h \leq d_2 \frac{r^{1-\alpha}}{4(1+r^\alpha)}$$

# Distance Bound

- Distance to the infinite line



$$r = \frac{d_1}{d_2}$$

$$h \leq d_2 \frac{r^{1-\alpha}}{4(1+r^\alpha)}$$

$$\alpha < 1/2 \quad h \leq \infty$$

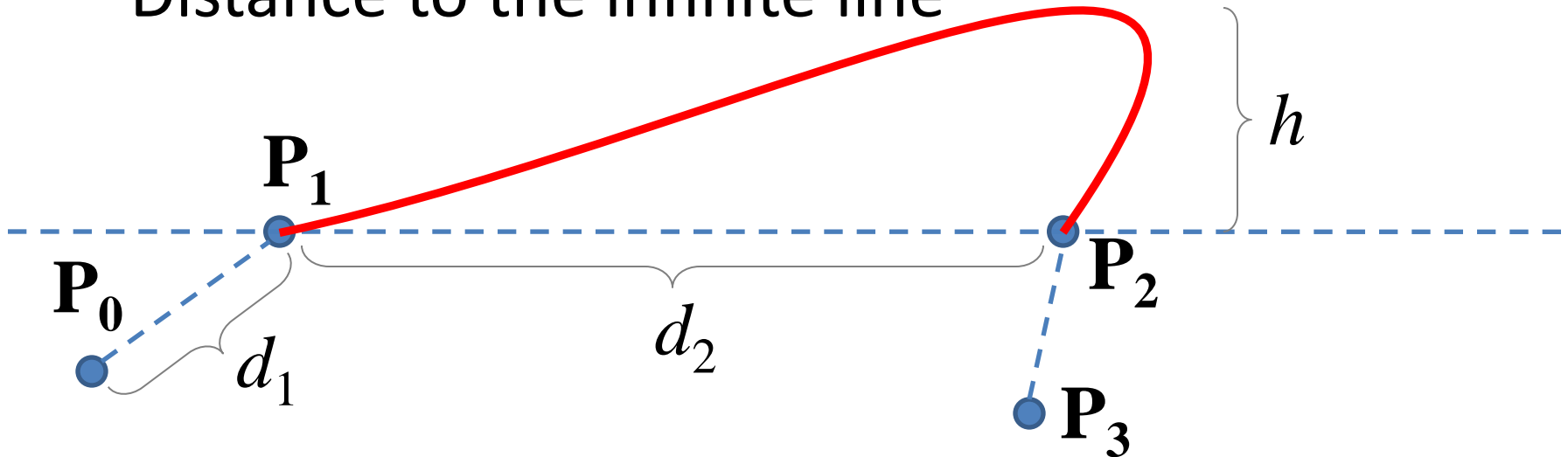
$$\alpha = 1/2 \quad h \leq d_2/4$$

$$\alpha = 2/3 \quad h \leq d_2/8$$

$$\alpha = 1 \quad h \leq d_2/4$$

# Distance Bound

- Distance to the infinite line



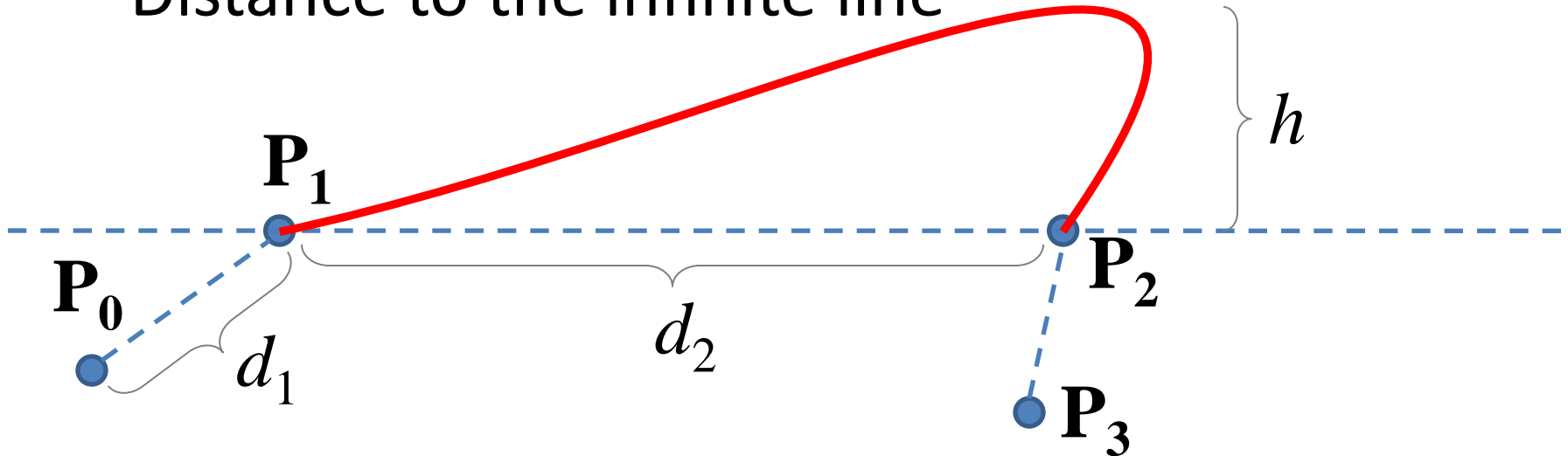
$$r = \frac{d_1}{d_2}$$

$$h \leq d_2 \frac{r^{1-\alpha}}{4(1+r^\alpha)}$$

$\alpha < 1/2$	$h \leq \infty$
$\alpha = 1/2$	$h \leq d_2/4$
$\alpha = 2/3$	$h \leq d_2/8$
$\alpha = 1$	$h \leq d_2/4$

# Distance Bound

- Distance to the infinite line



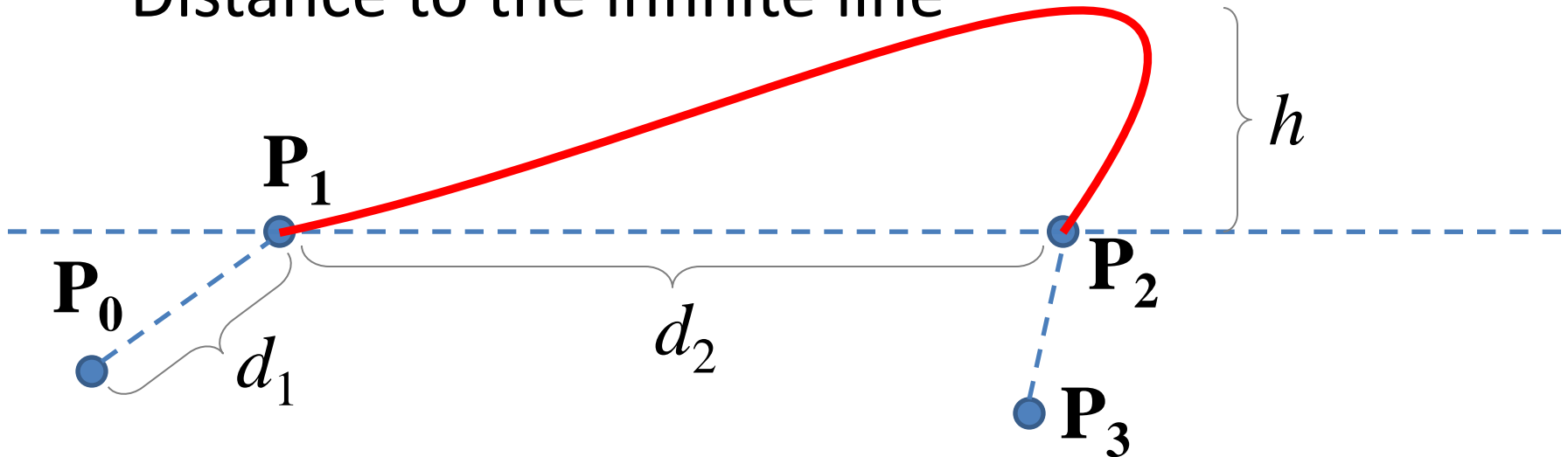
$$r = \frac{d_1}{d_2}$$

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# Distance Bound

- Distance to the infinite line



$$r = \frac{d_1}{d_2}$$

$$h \leq d_2 \frac{r^{1-\alpha}}{4(1+r^\alpha)}$$

$$\alpha < 1/2 \quad h \leq \infty$$

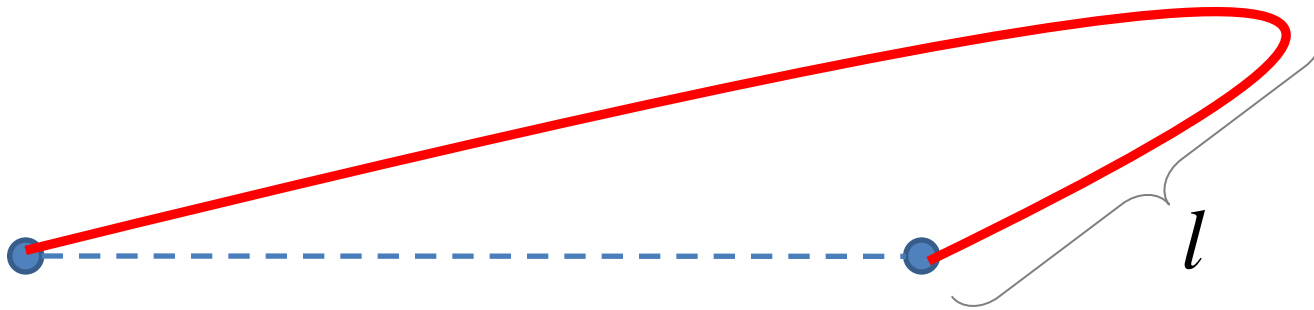
$$\alpha = 1/2 \quad h \leq d_2/4$$

$$\alpha = 2/3 \quad h \leq d_2/8$$

$$\alpha = 1 \quad h \leq d_2/4$$

# Distance Bound

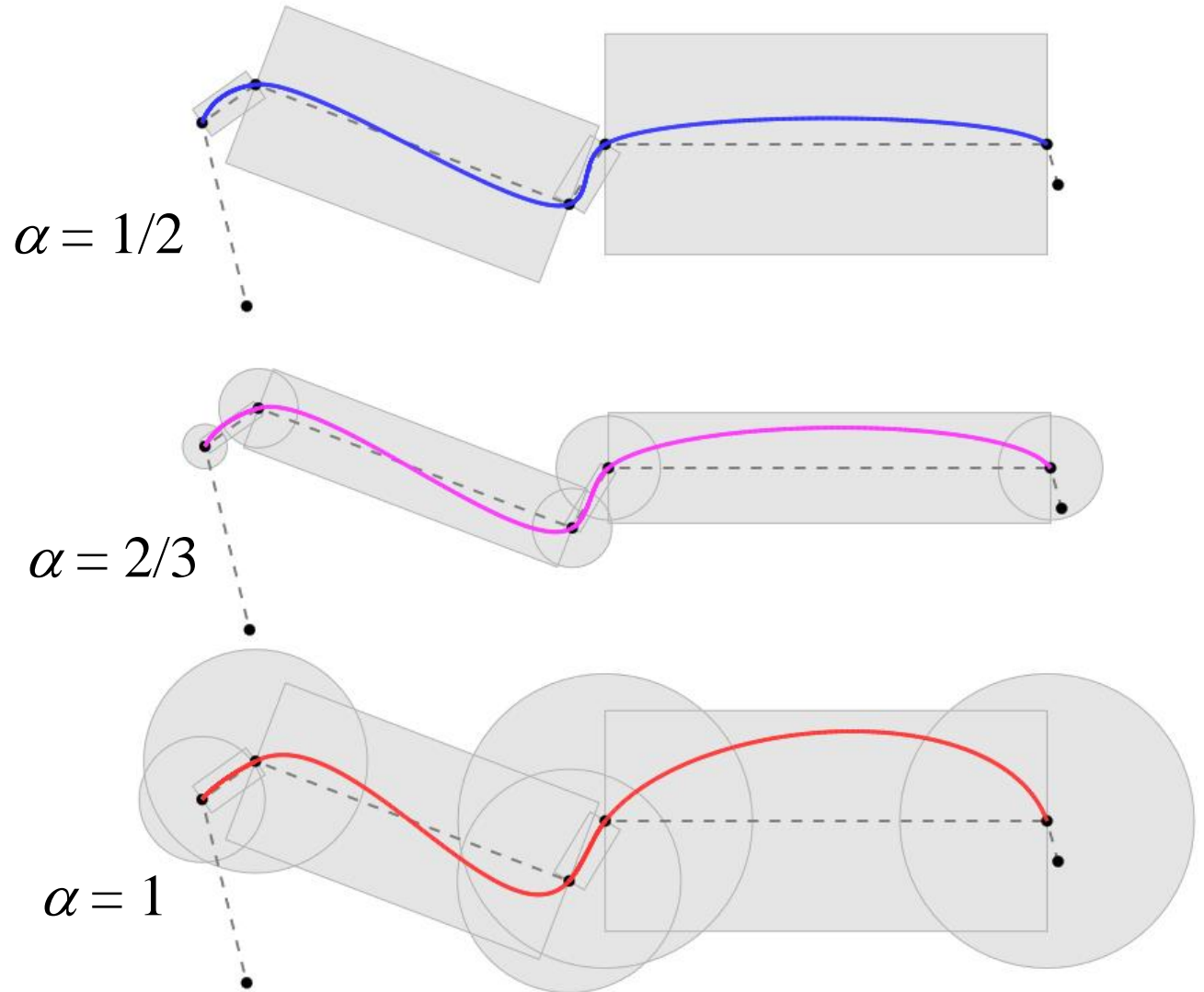
- Distance to the end points



$$l \leq \frac{d_2 \sqrt{r^2 - r^{4\alpha}}}{3 r^\alpha (1 + r^\alpha)}$$

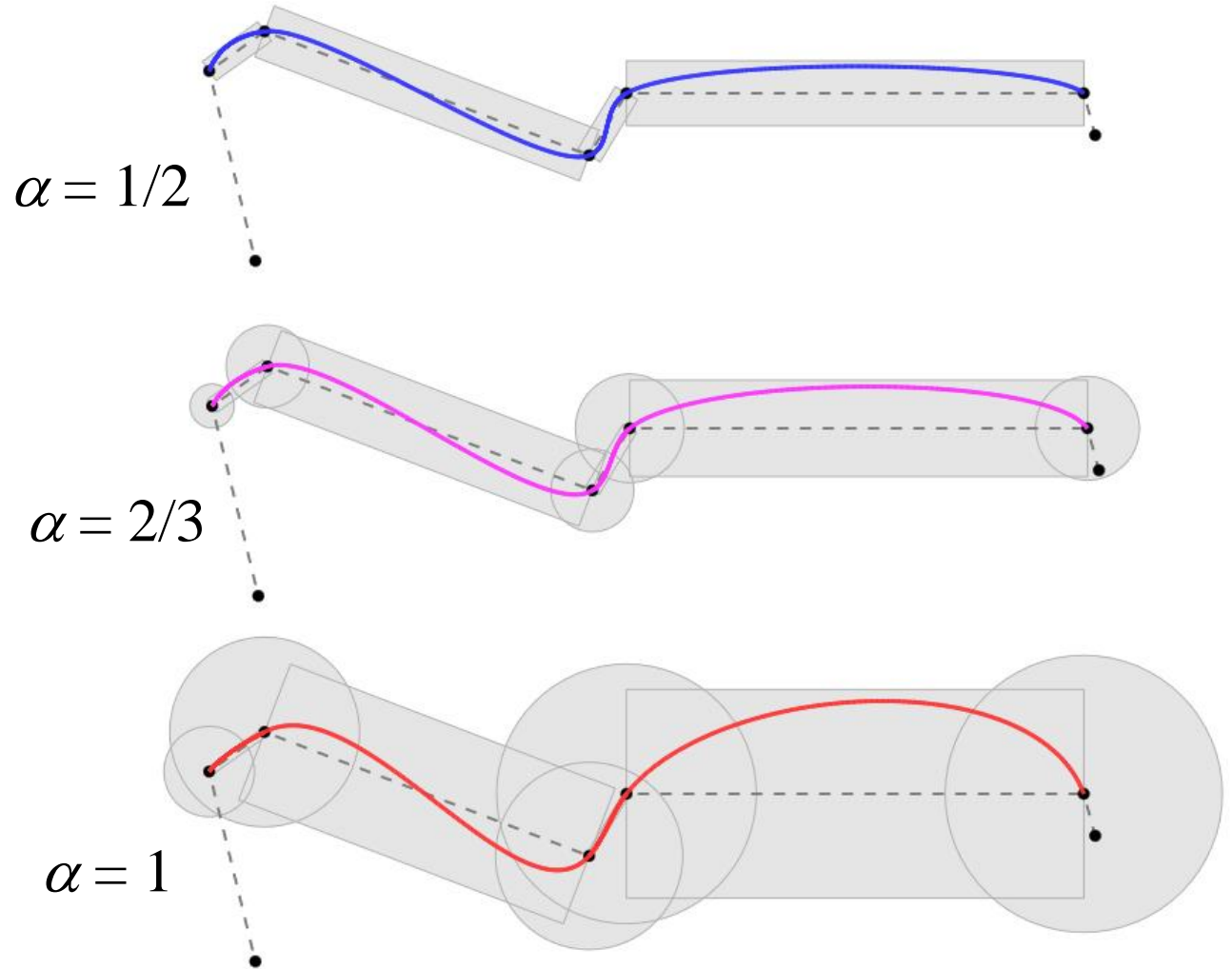
# Distance Bound

$\alpha$  only



# Distance Bound

$\alpha$  and  $r$

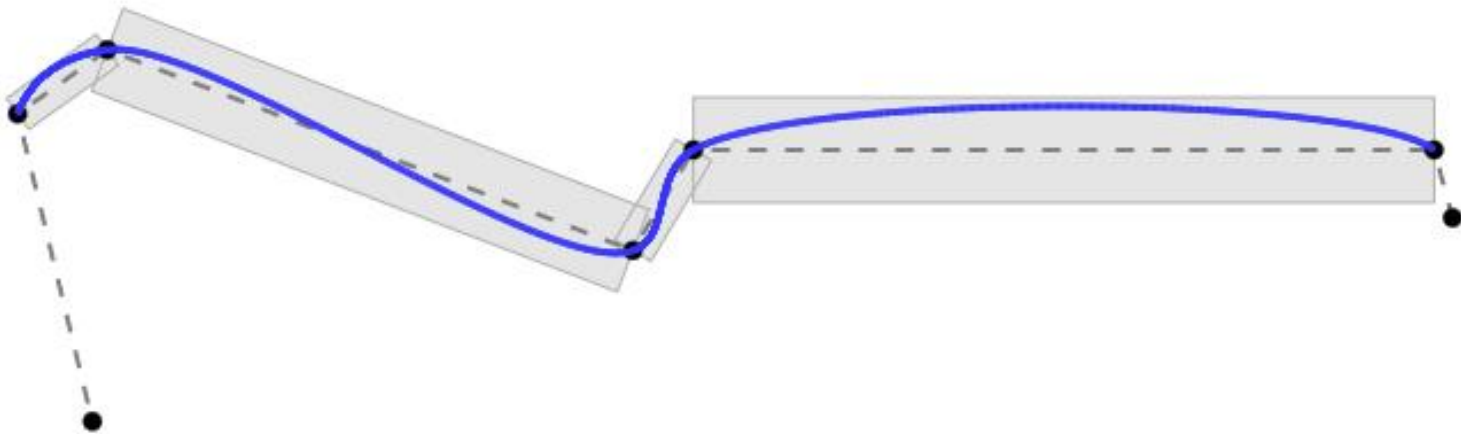




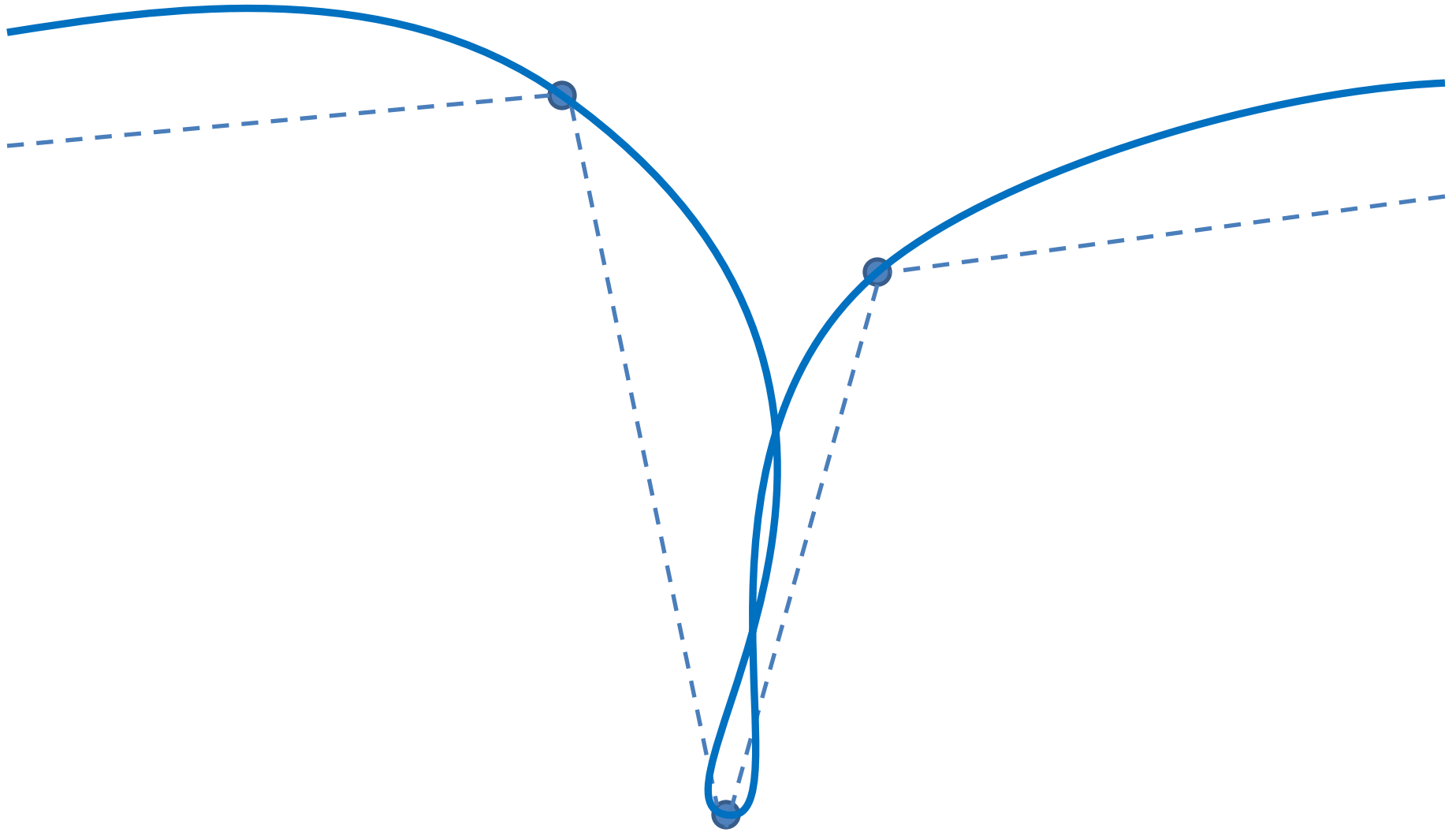
On the Parameterization of Catmull-Rom Curves

# **INTERSECTION-FREE CURVES**

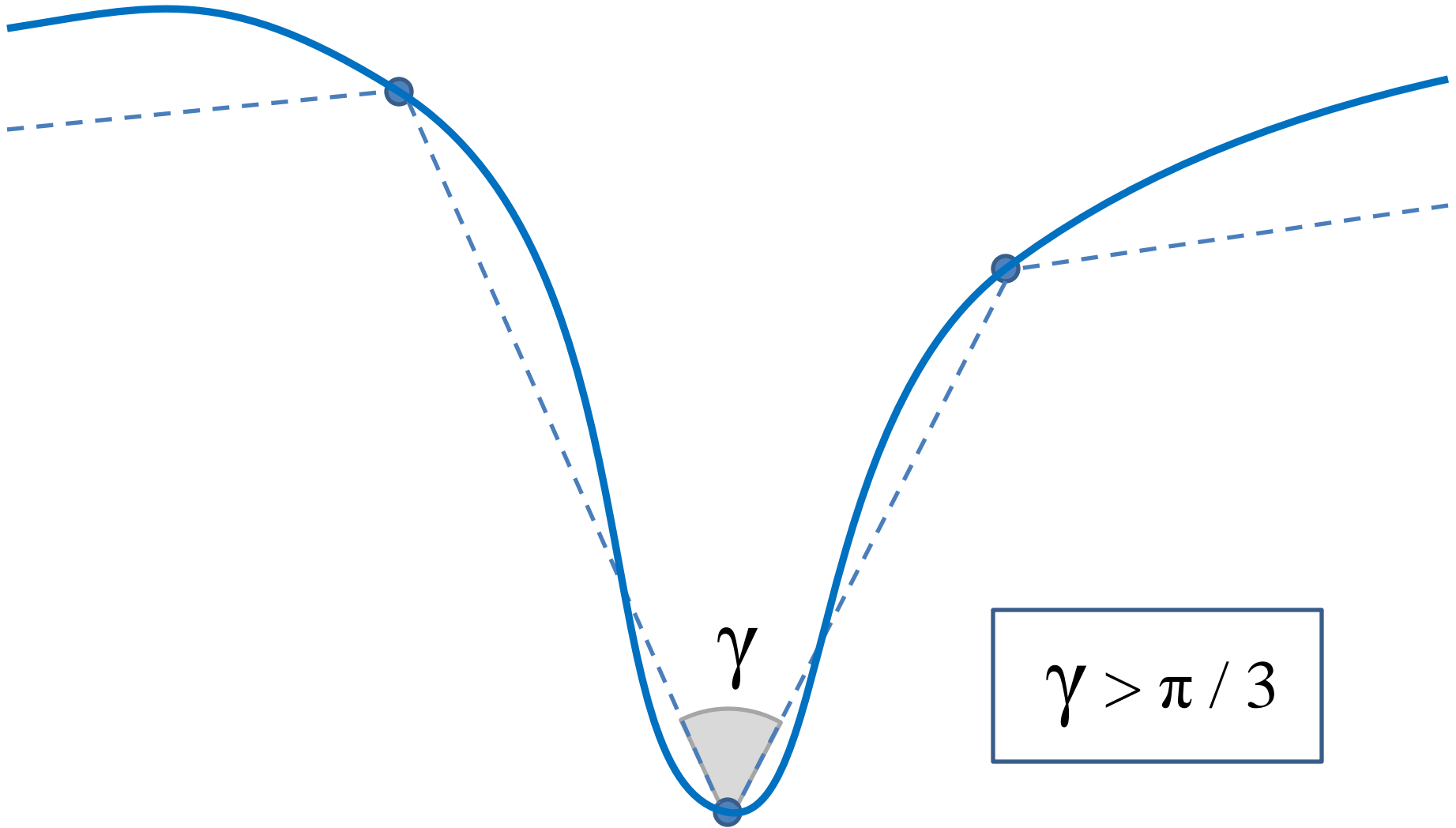
# Intersection-Free Curves



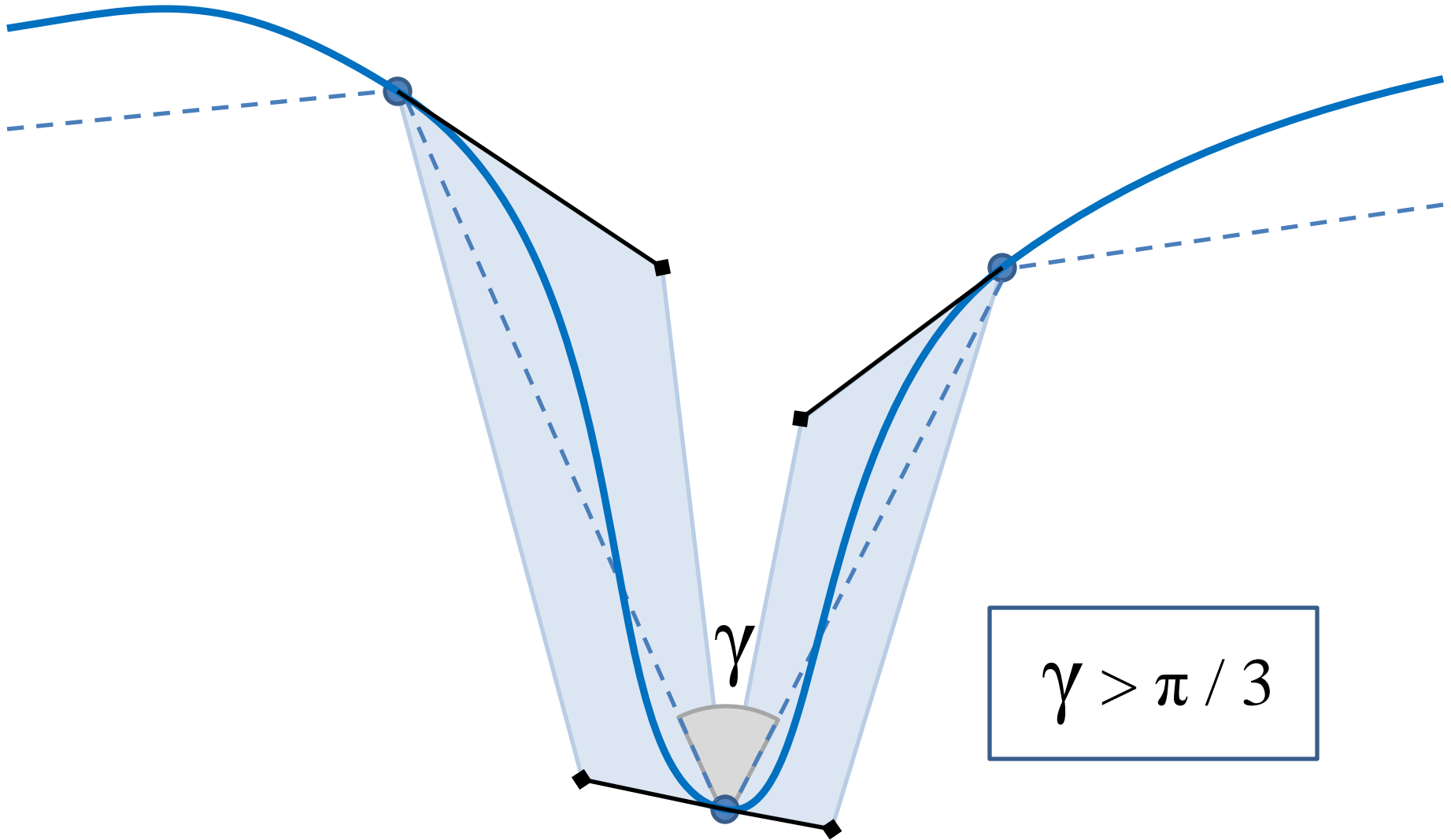
# Intersection-Free Curves



# Intersection-Free Curves



# Intersection-Free Curves



# Intersection-Free Curves

- Avoid self-intersections
  - Centripetal parameterization
- Avoid adjacent segment intersections
  - Control polygon angle  $> \pi / 3$
- Avoid non-adjacent segment intersections
  - Bounding box

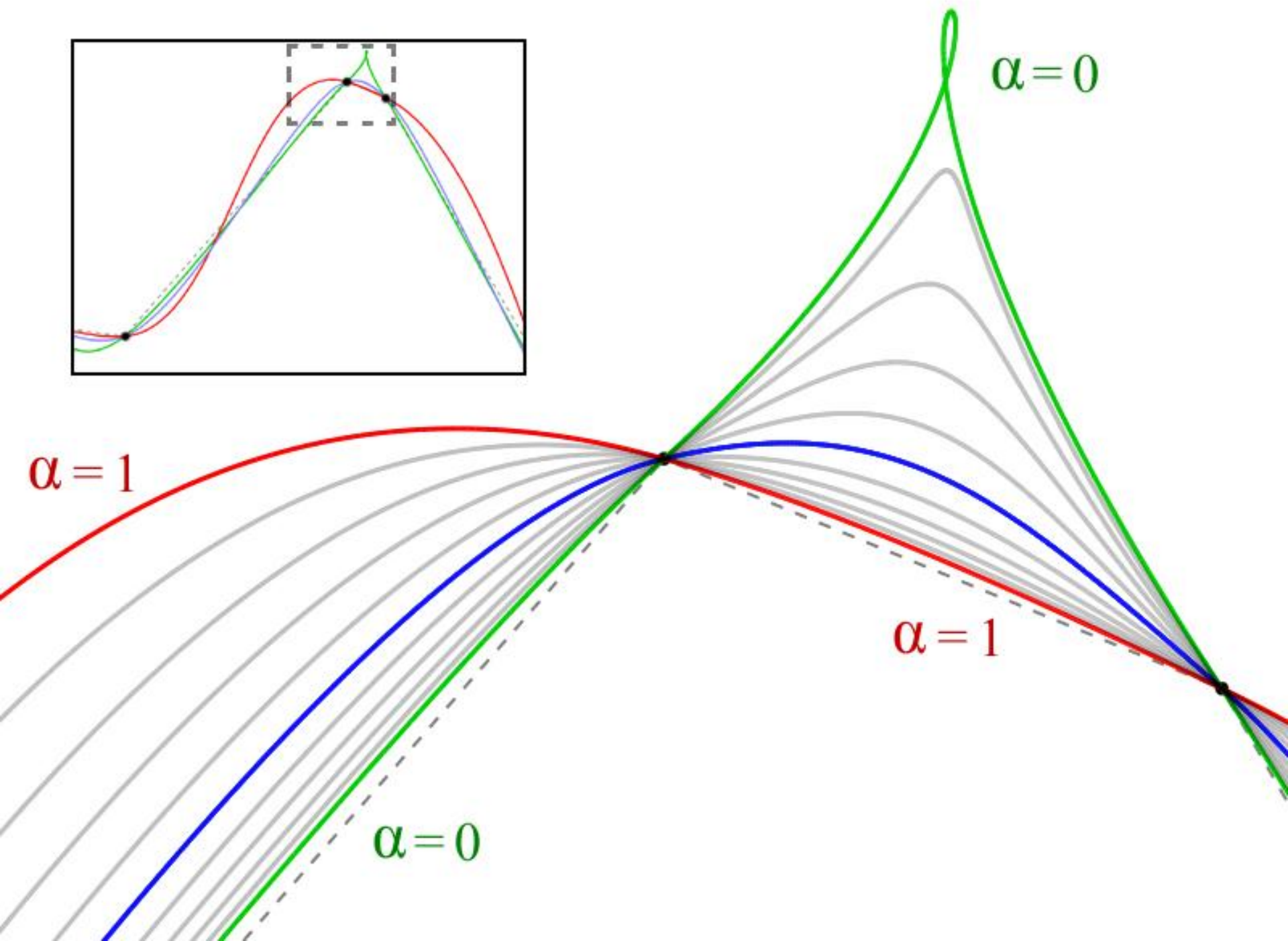
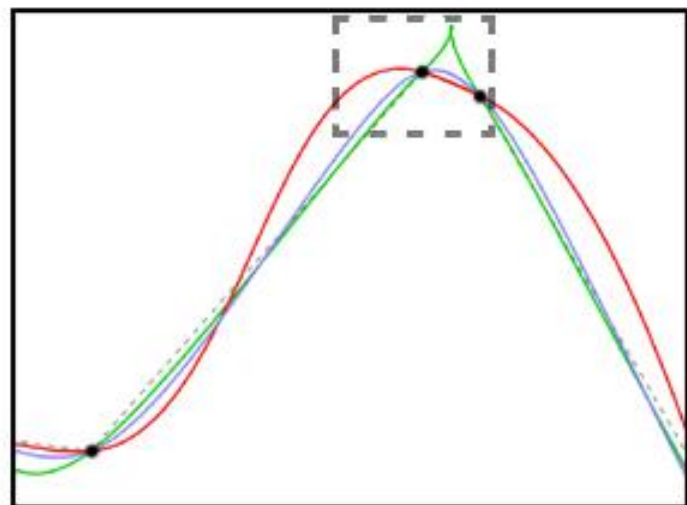
On the Parameterization of Catmull-Rom Curves

# **DISCUSSION**

# Discussion

- Distance to Control Polygon
  - Uniform is closer for longer segments
  - Chordal is closer for shorter segments



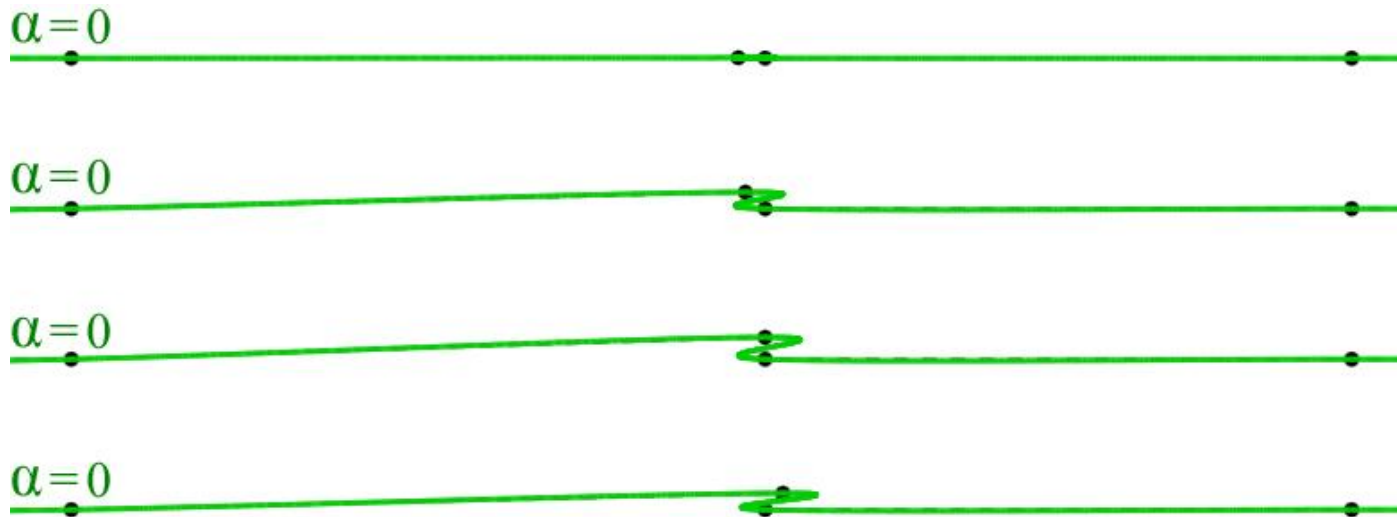


# Discussion

- Edge Direction
  - Chordal has extreme sensitivity to short edge directions

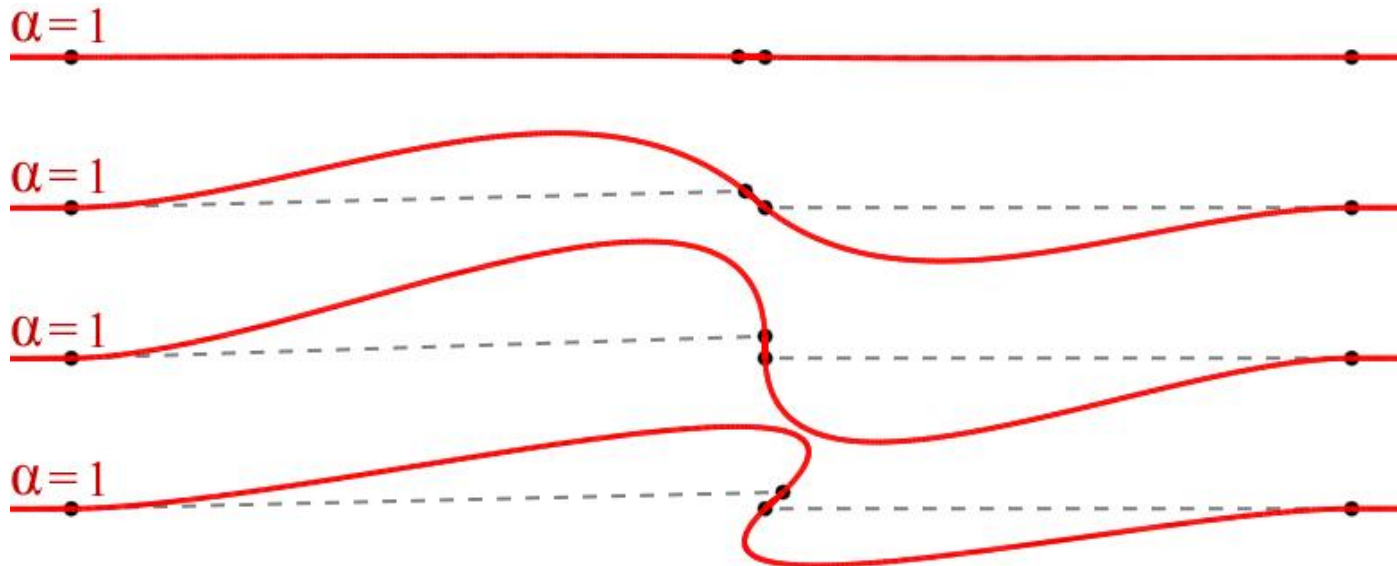
# Discussion

- Edge Direction
  - Chordal has extreme sensitivity to short edge directions



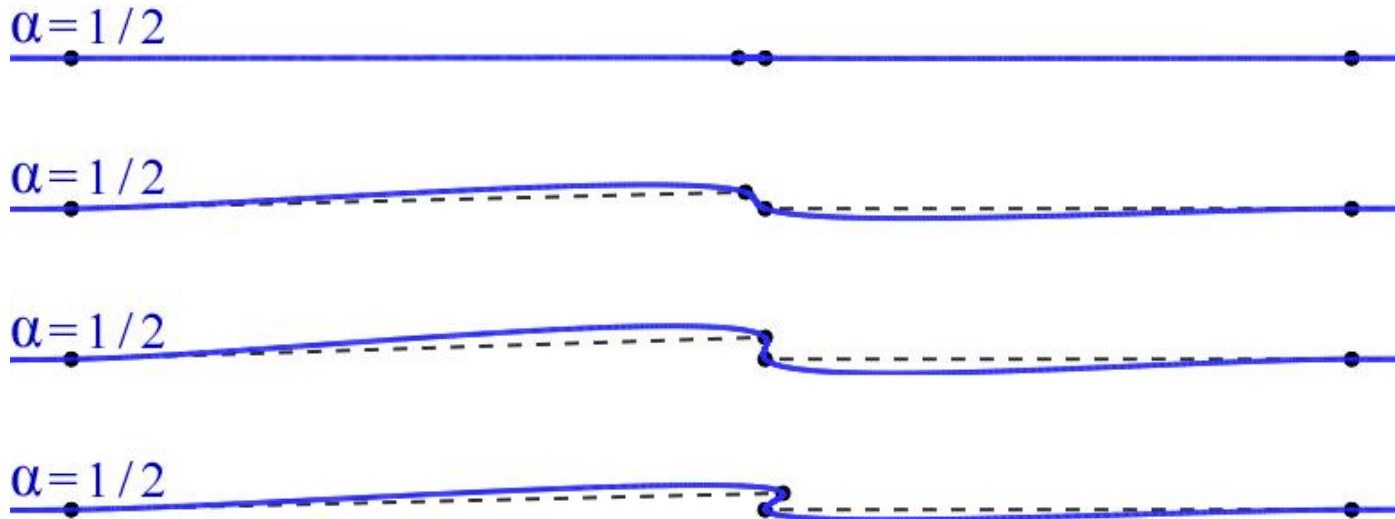
# Discussion

- Edge Direction
  - Chordal has extreme sensitivity to short edge directions



# Discussion

- Edge Direction
  - Chordal has extreme sensitivity to short edge directions



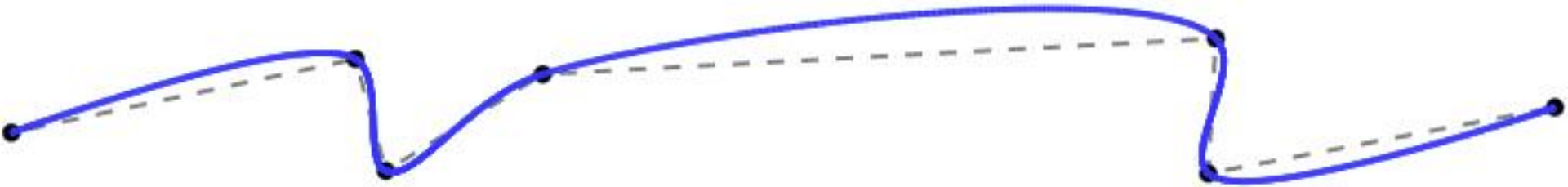
# Discussion

- Curvature
  - Centripetal *tends* to have higher curvature at control points.



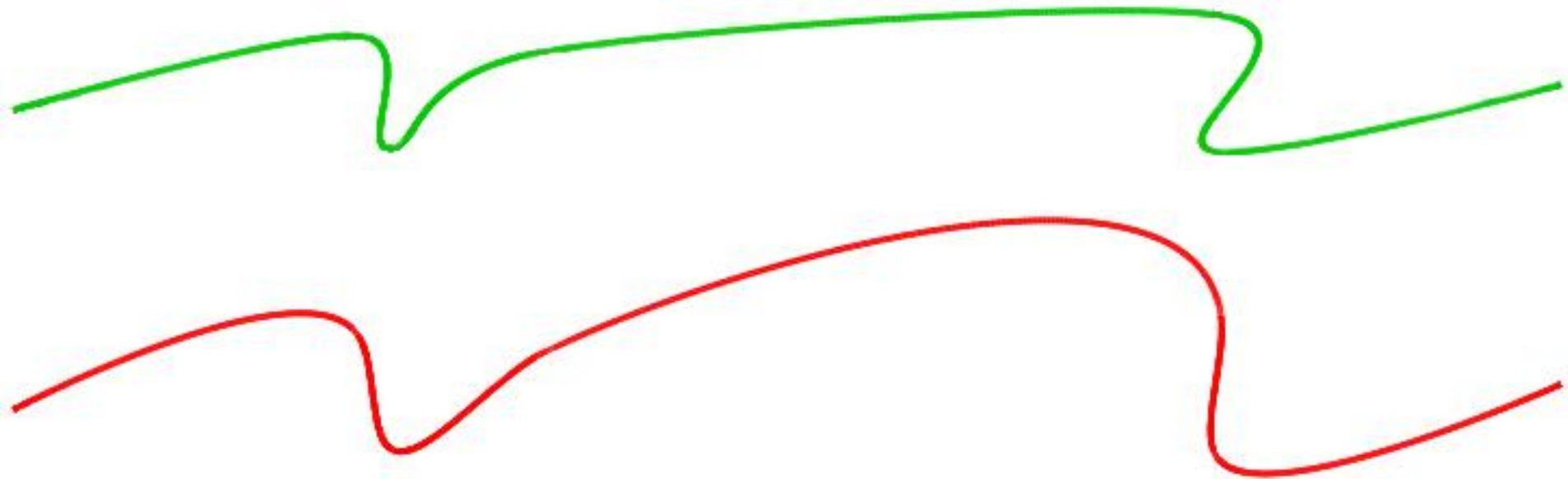
# Discussion

- Curvature
  - Centripetal *tends* to have higher curvature at control points.



# Discussion

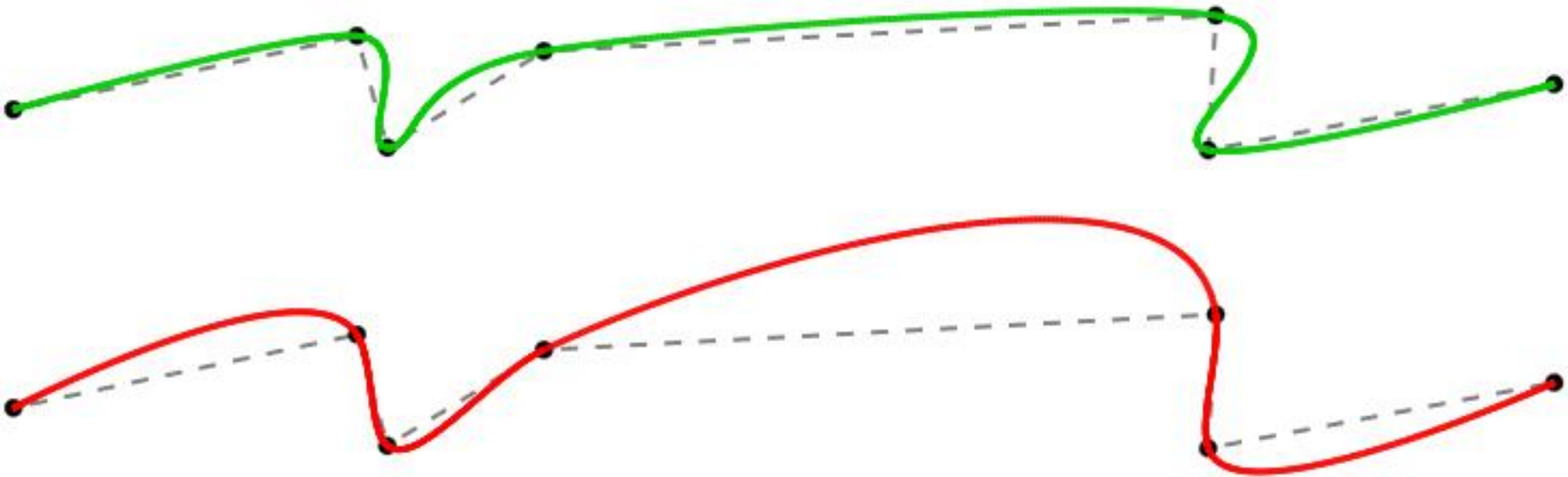
- Curvature
  - Centripetal *tends* to have higher curvature at control points.





# Discussion

- Curvature
  - Centripetal *tends* to have higher curvature at control points.



# Catmull-Rom Curves



Lee Perry-Smith



Lee Perry-Smith

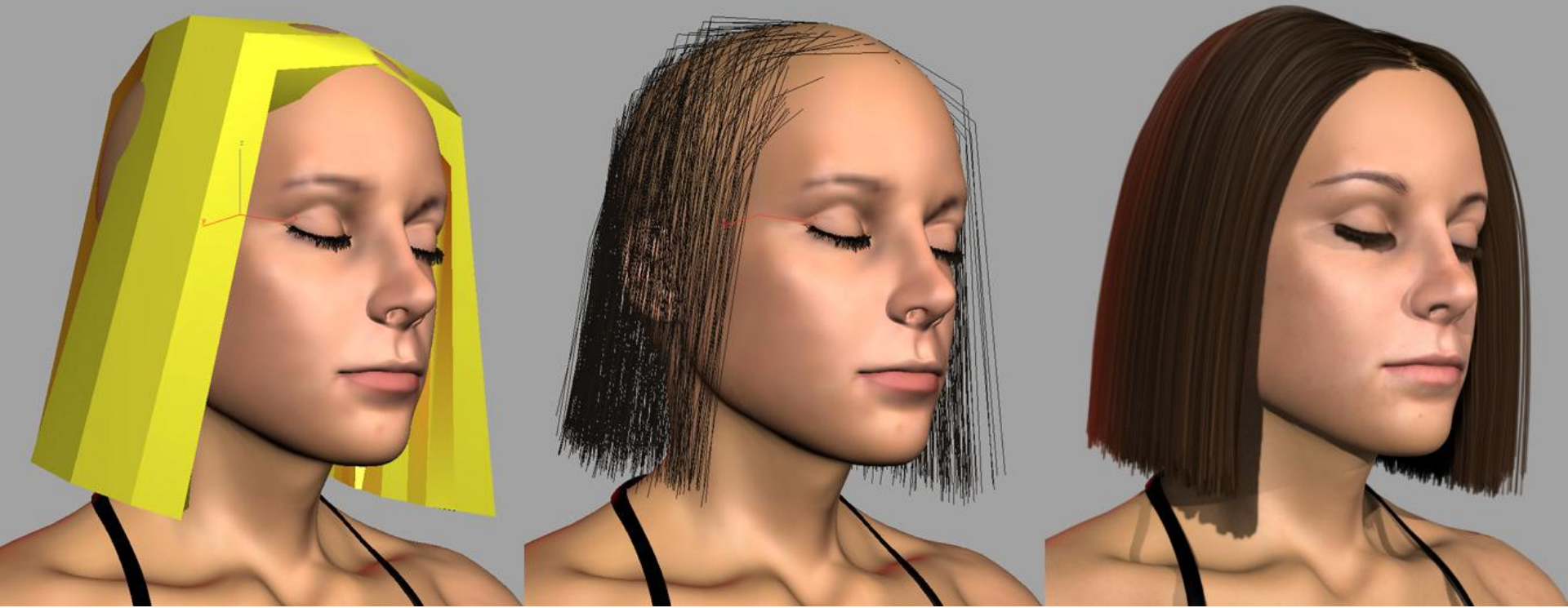


Alexander Tomchuk

Cem Yuksel, Scott Schaefer, John Keyser, "Hair Meshes," Siggraph Asia 2009

**HAIRFARM**<sup>™</sup>  
THE ULTIMATE HAIR PLUG-IN FOR 3DS MAX

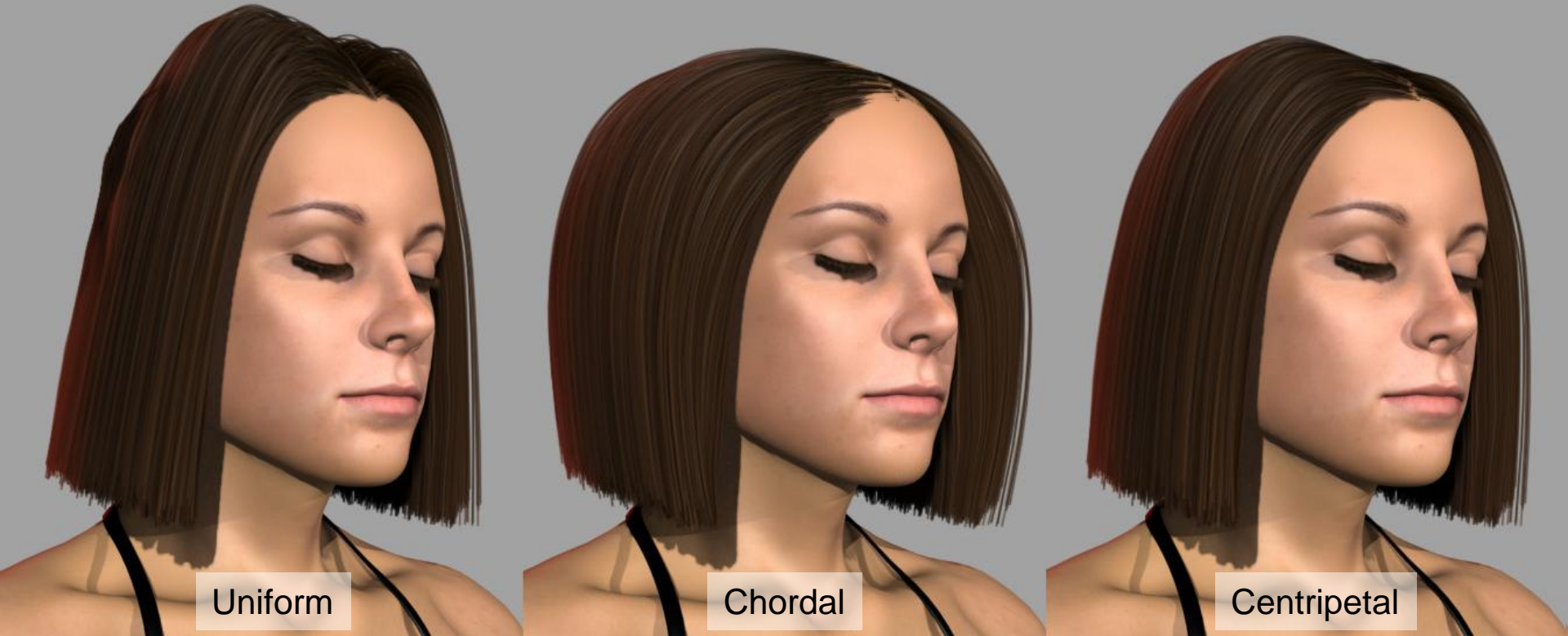
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# Summary

- Parameterization of Catmull-Rom curves

$$0 \leq \alpha \leq 1$$

- Cusps and self-intersections
- Distance bound
- Intersection-free curves
  
- $C^1$  Catmull-Rom curves only!

On the Parameterization of Catmull-Rom Curves

**QUESTIONS?**